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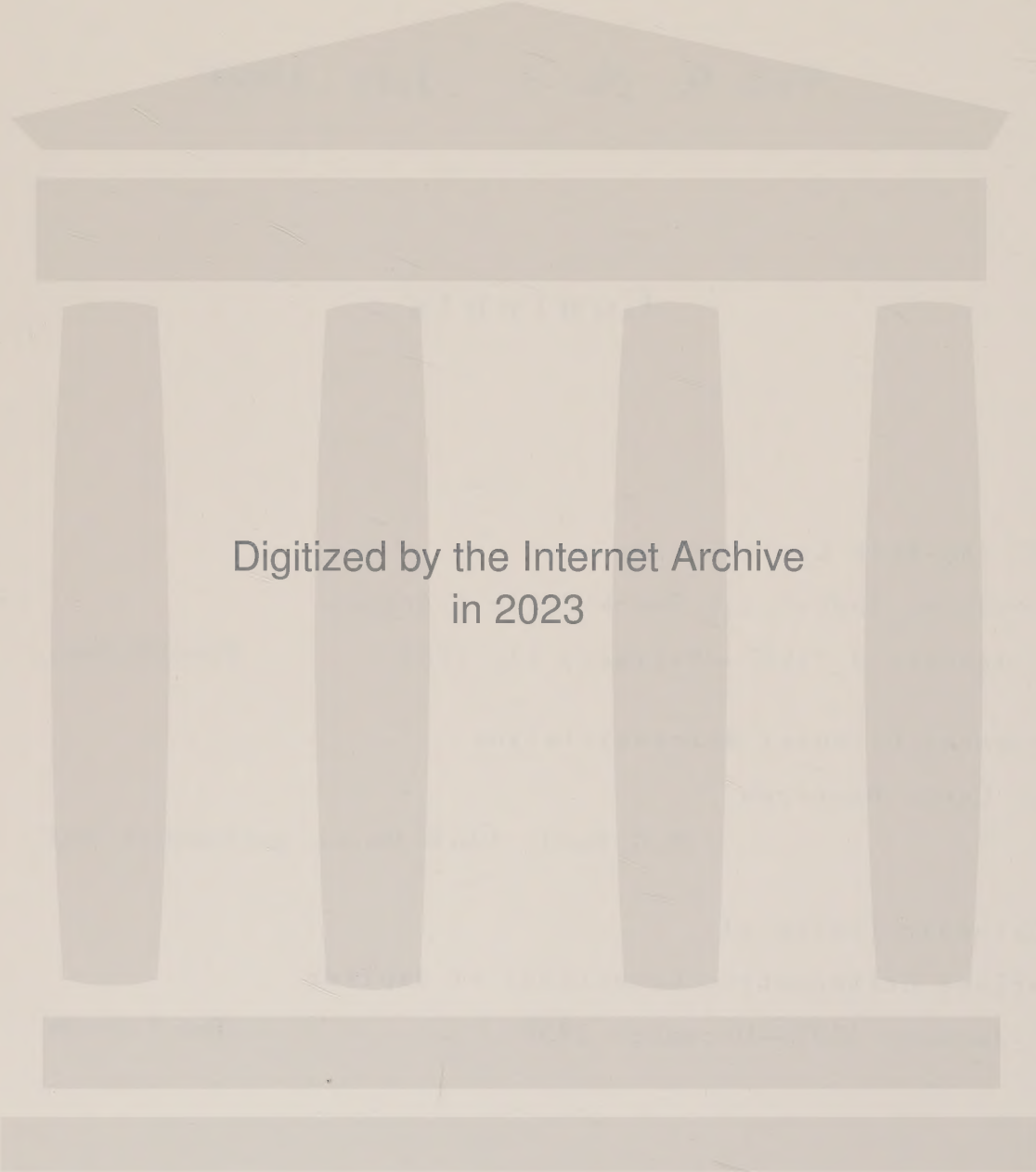
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THE 17.166 - WEEK CYCLE IN THE DOW-JONES INDUSTRIAL STOCK PRICE AVERAGES JANUARY 9, 1897 — FEBRUARY 13, 1960

BY EDWARD R. DEWEY

ABSTRACT

An earlier study relative to a four-month cycle in common stock prices is continued backward from 1928 through 1897. The cycle is found to persist during the 31 additional years. It would be hard for a cycle as persistent as this one to be present merely as a result of chance, or as the result of free oscillation.

The length of the cycle proves to be 17.166 weeks (120.166 days) plus or minus perhaps .015 weeks (.1 day). On a rigid length basis, an ideal time of crest proves to be 1/5 of a week into the week ending July 30, 1960. Other highs are, ideally, at 17.166-week intervals prior to that time, and, if the cycle continues, forward from that time.

On the average, the cycle has a strength of 100.60% of trend at ideal time of crest, 99.24% of trend at ideal time of trough. This variation amounts to 1.36% in half a wave (8.583 weeks); 2.72% in one round trip; and cumulatively, to 8.16% in a year.

But the wave length is not perfectly regular. On the average, it is alternately a shade longer or a shade shorter than $17 \frac{1}{6}$ weeks. This circumstance makes the crests (and troughs) come first early, then late. This alternation of wave length takes place at about 72-cycle (23- or 24-year) intervals.

Shorter variations of wave length and seeming rhythmic variations of amplitude are noted but are not studied.

It is possible that the average 17.166-week wave present on the average in these figures is a vestige of a much stronger and somewhat irregular disturbance that does, however, have an underlying tendency toward regularity.

The wave length of the $17 \frac{1}{6}$ -week cycle is not found to be related to the wave length of any other well-established cycle.

No cause for the 17.166-week cycle is discovered, but some possible causes that might be conjectured are disproved.

In view of the fact that stock prices are available weekly—even daily for five days out of every week if such detail should be required—it was decided to study with considerable thoroughness one of the cycles that had been discovered in these figures.

In order to get extreme accuracy and to get so many repetitions of the cycle that the possibility of chance could be virtually ruled out, it was decided to study a comparatively

short cycle. The one chosen is the well-known one that has a wave length of a little over 17 weeks—about a third of a year.

The number of repetitions of a cycle of this length in a series of figures as long as this—63 years—not only enables us to measure the length of the cycle with great precision, but enables us to study variations in wave length and strength, insofar as these are present.

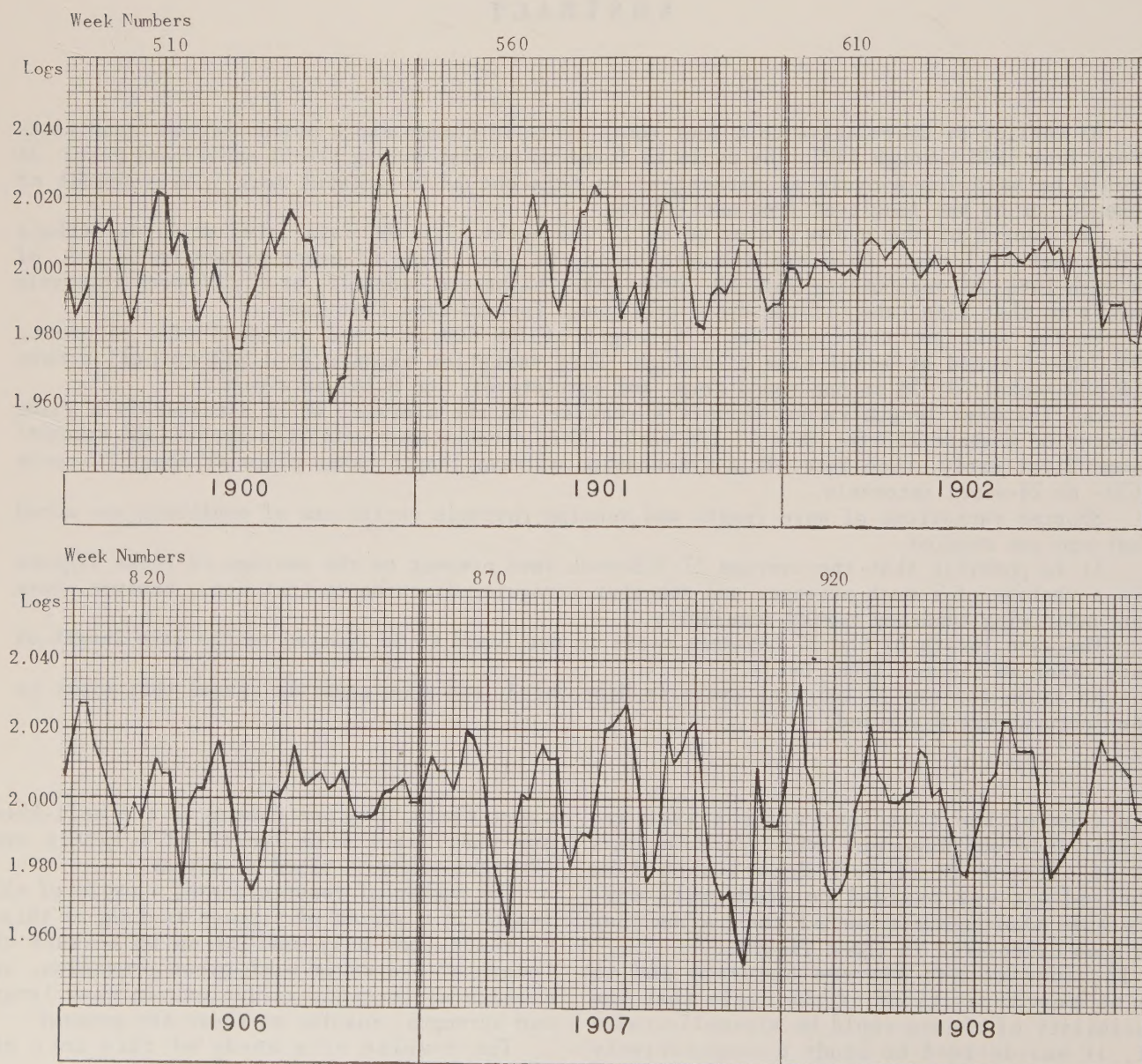
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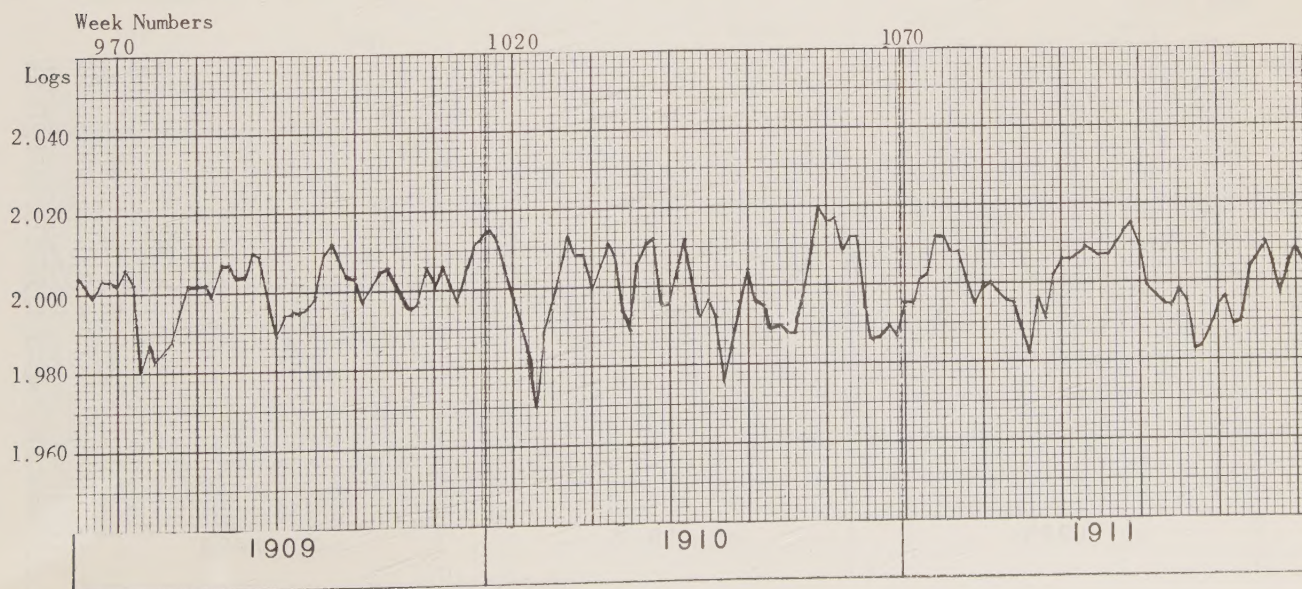
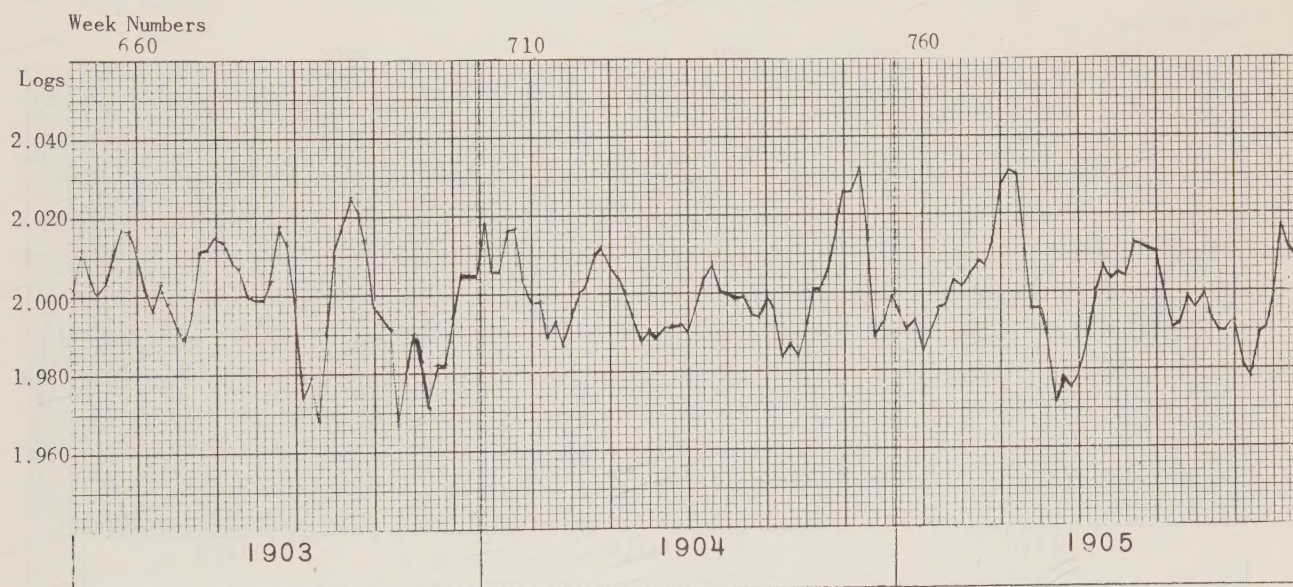
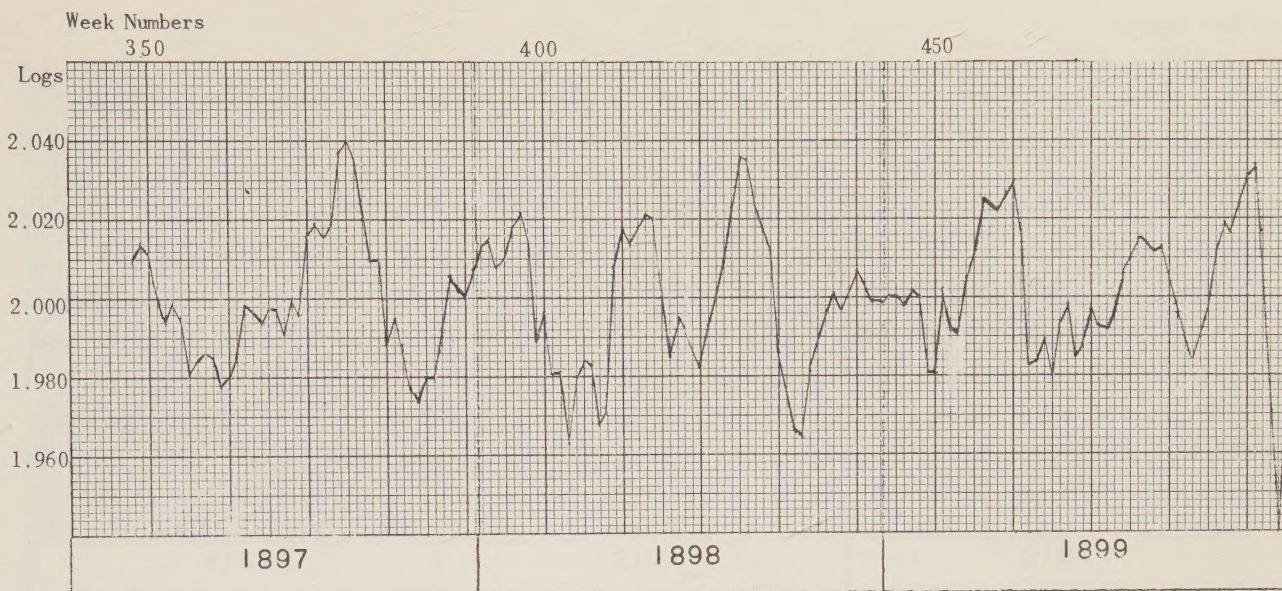
Fig. 1: DOW - JONES INDUSTRIAL STOCK PRICE AVERAGES

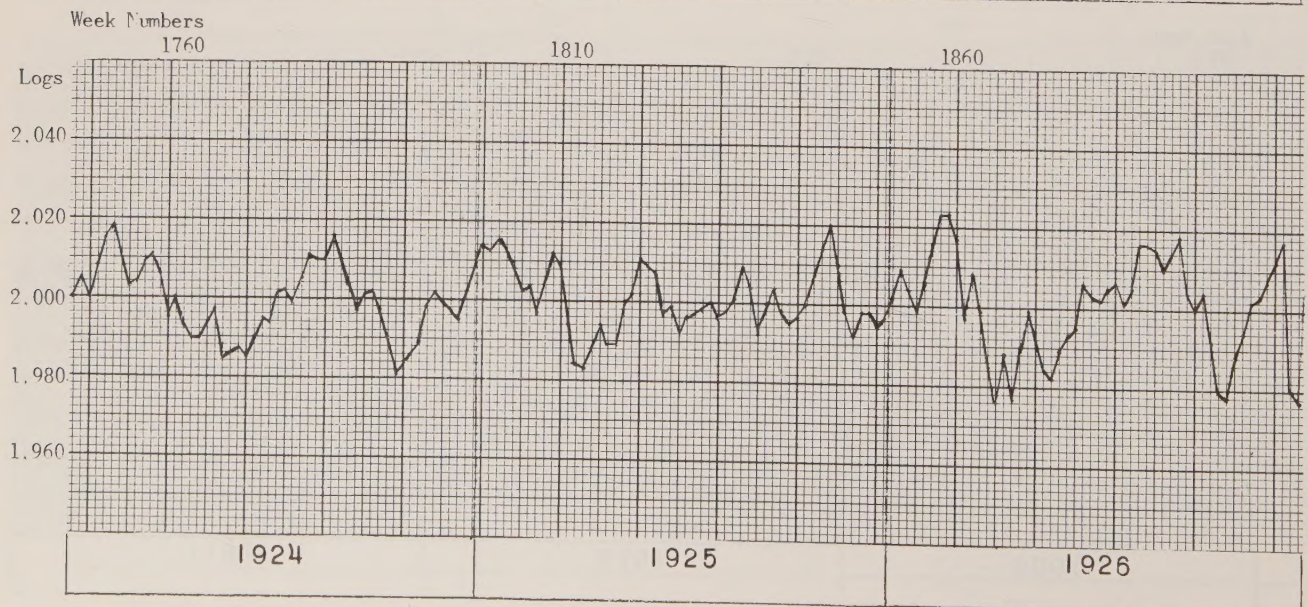
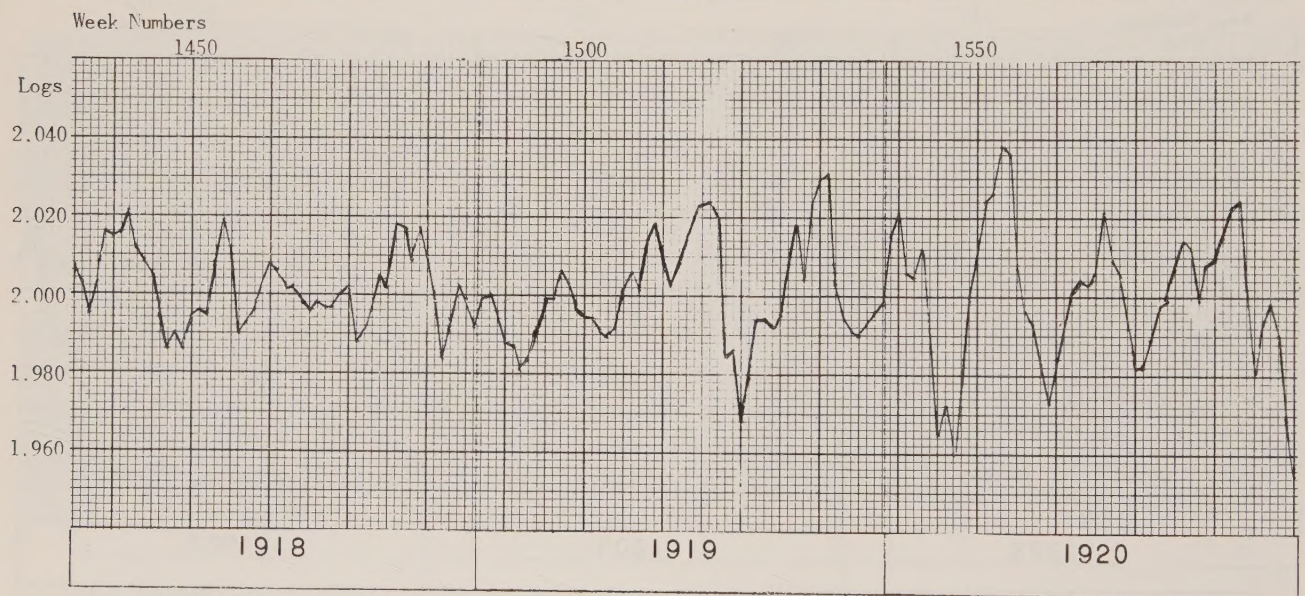
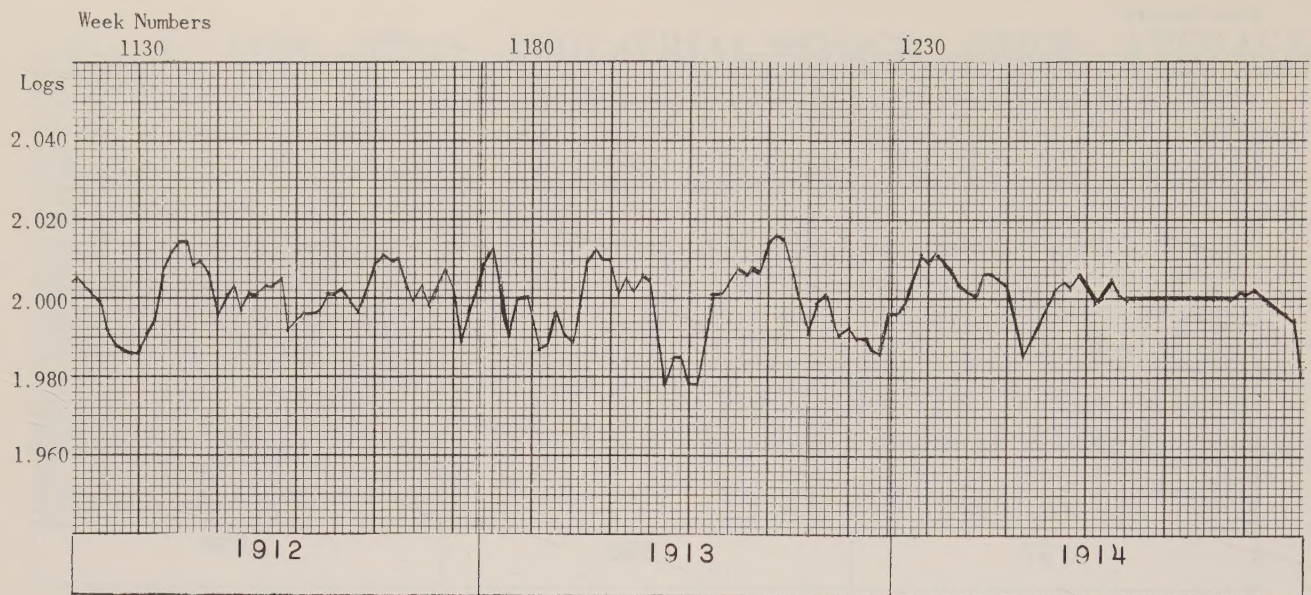
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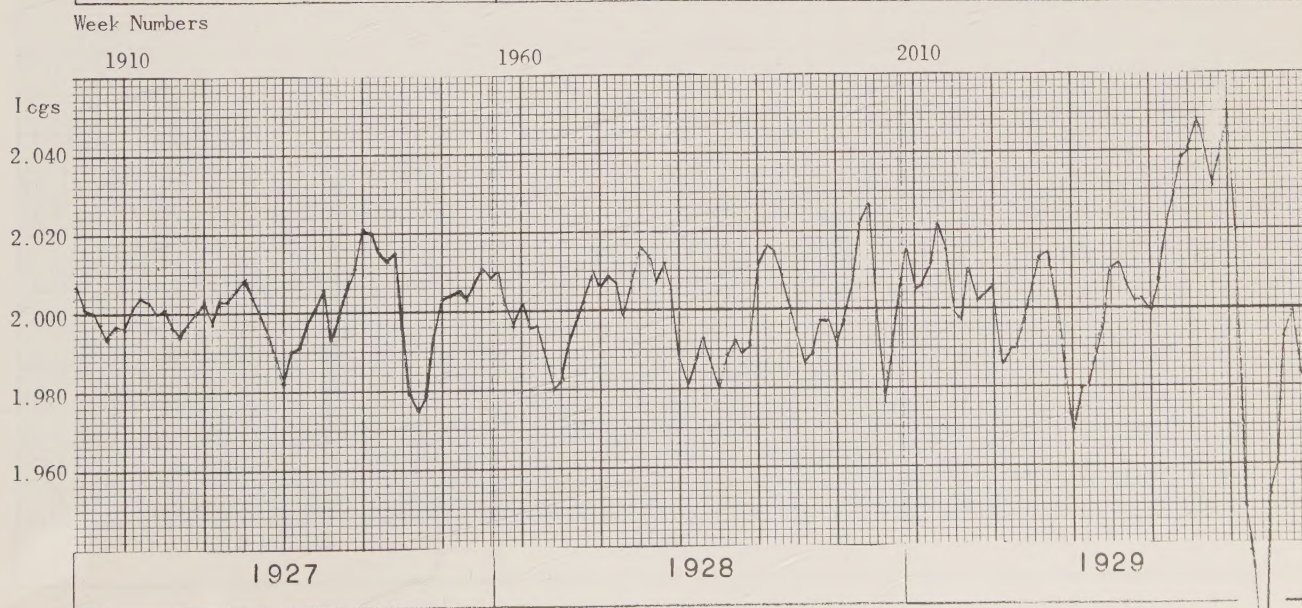
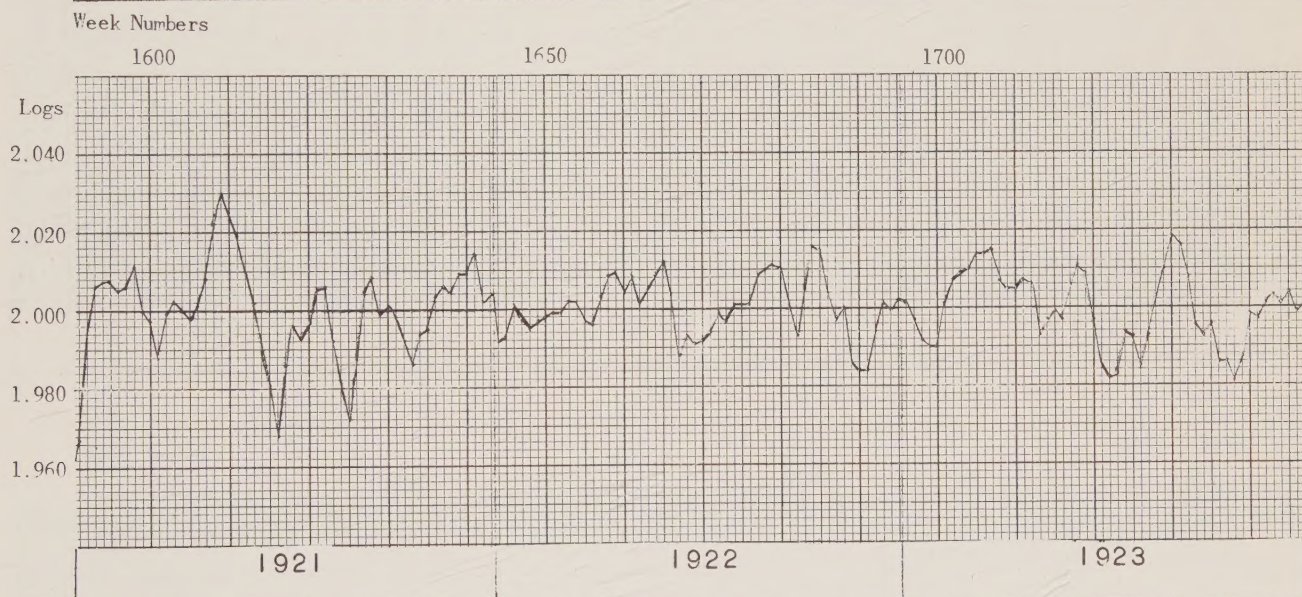
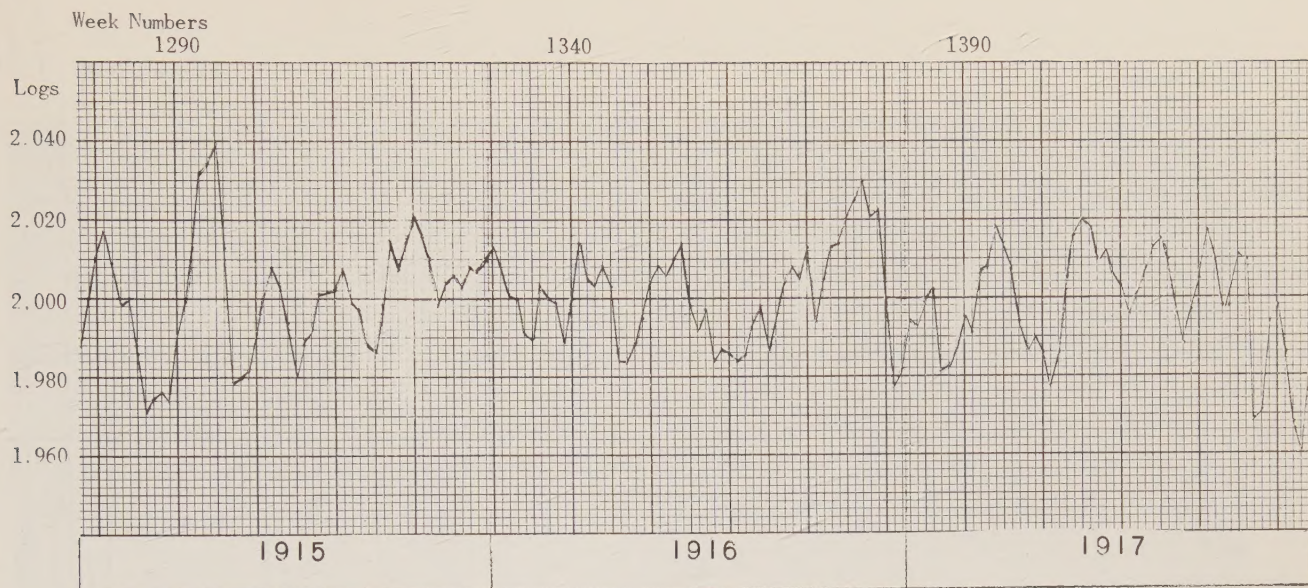
Through Week Ending December 19, 1959

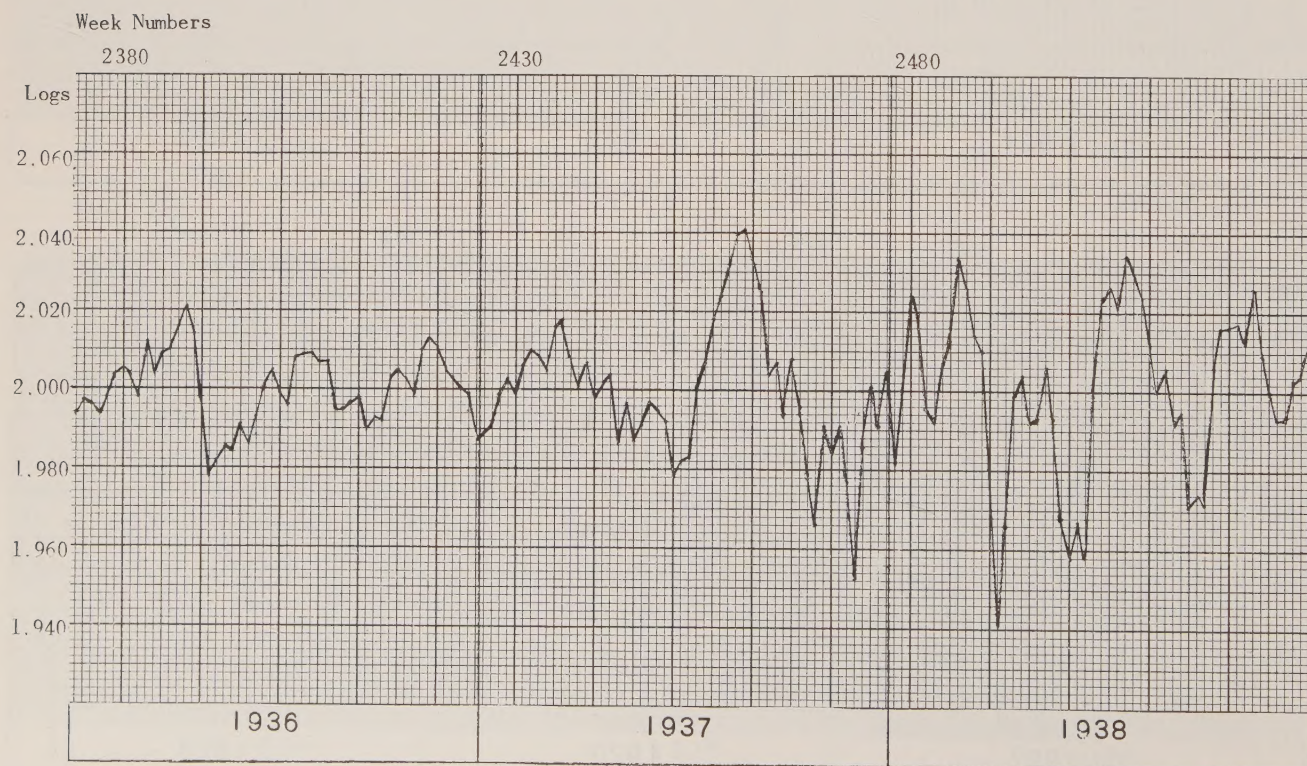
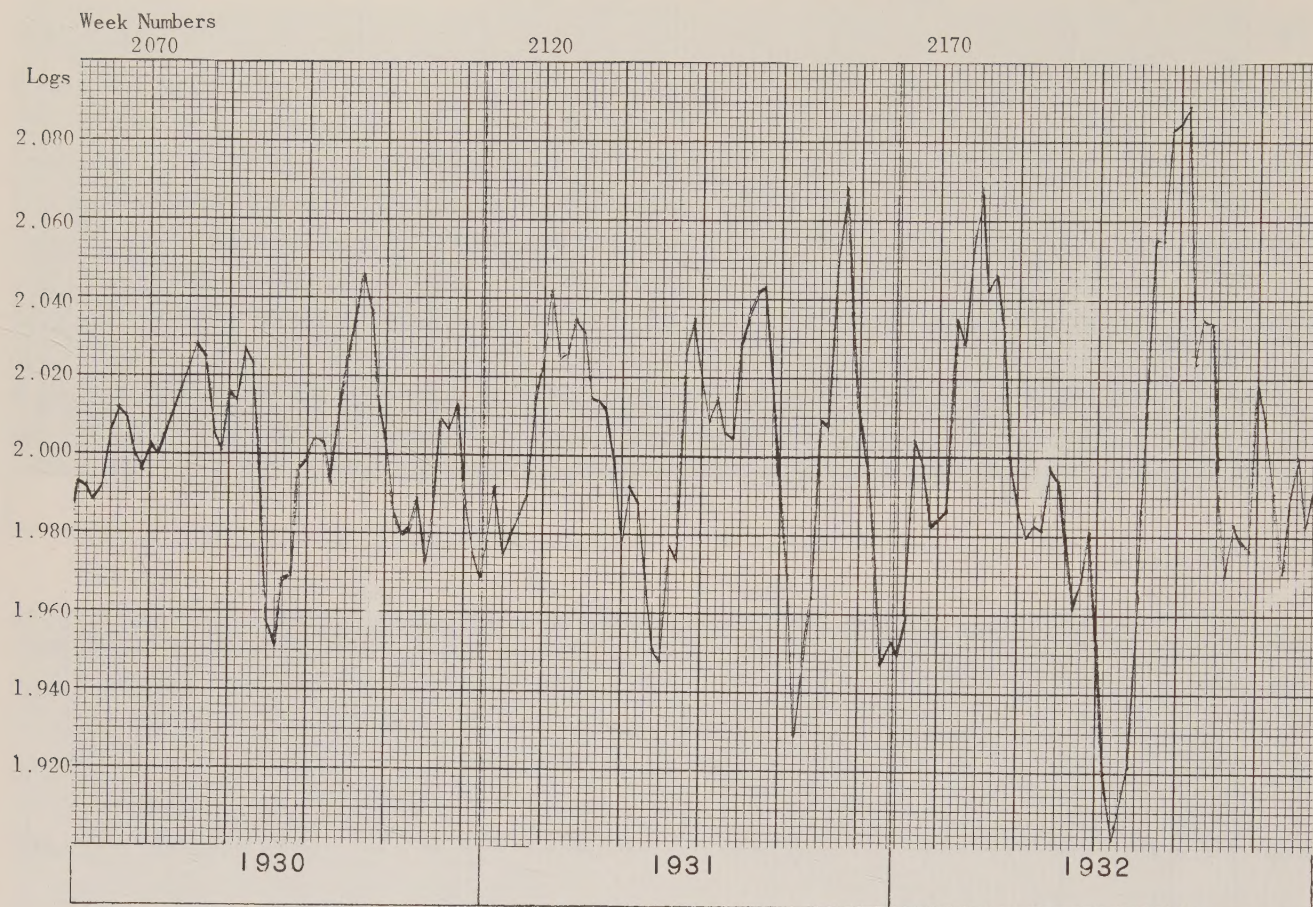
Expressed (In Logs) As Deviations
From Their 17-Week Moving Average Trend

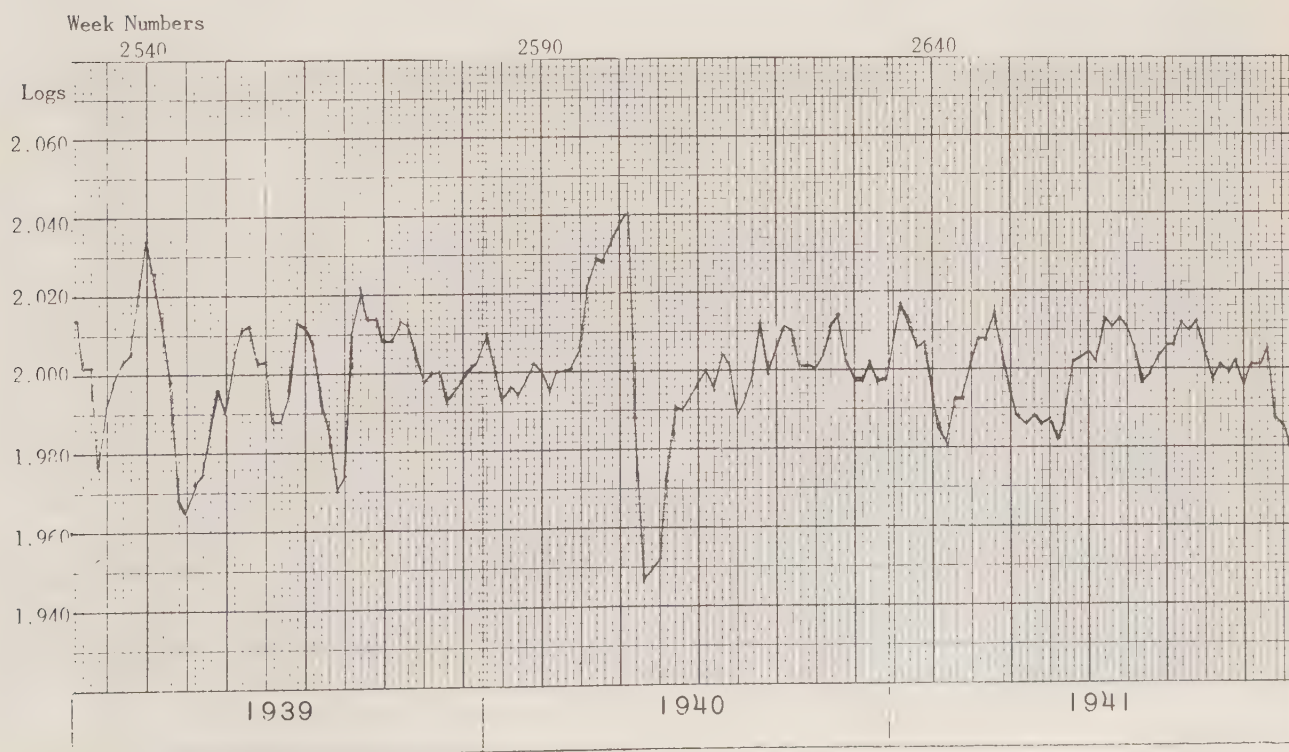
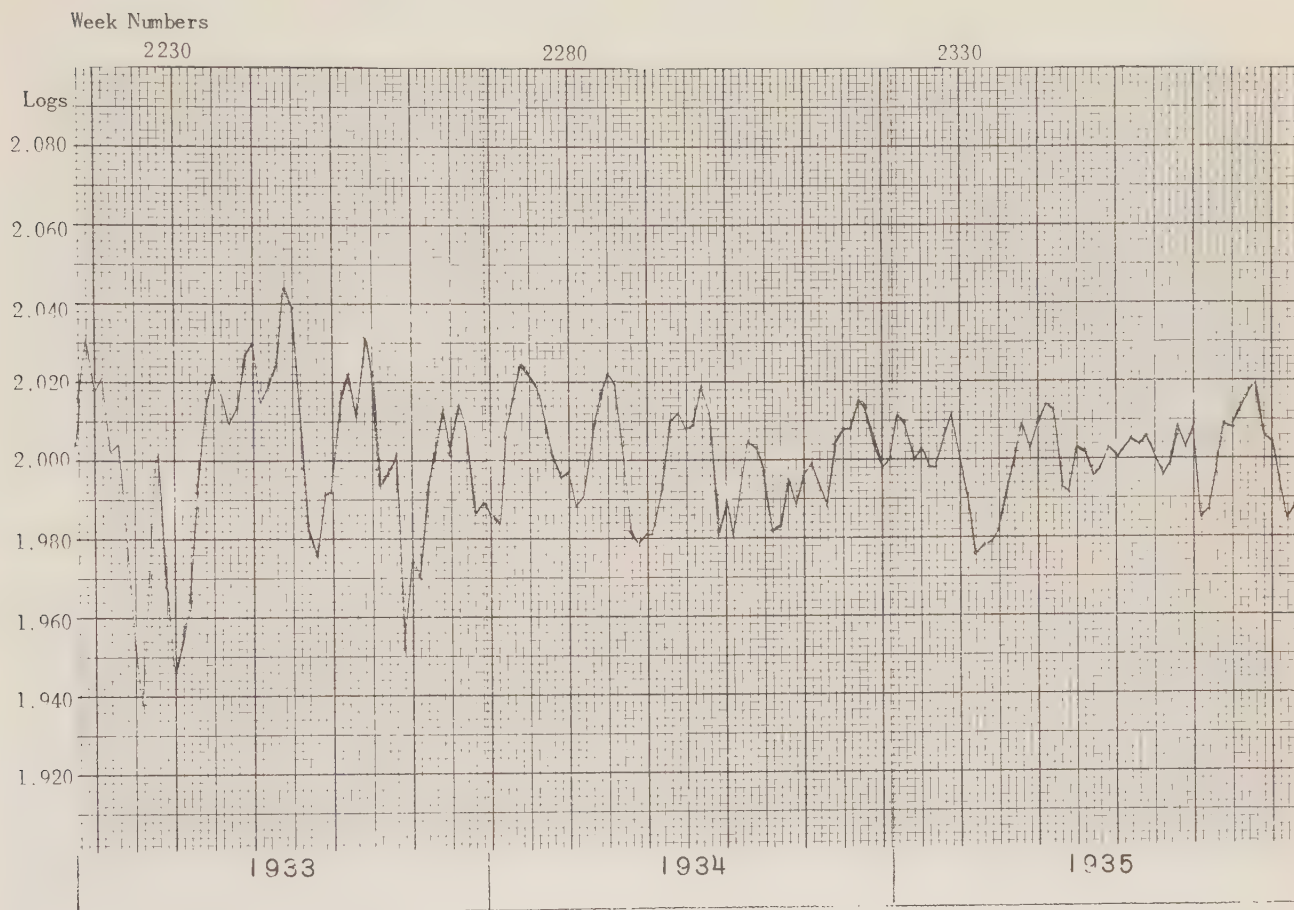


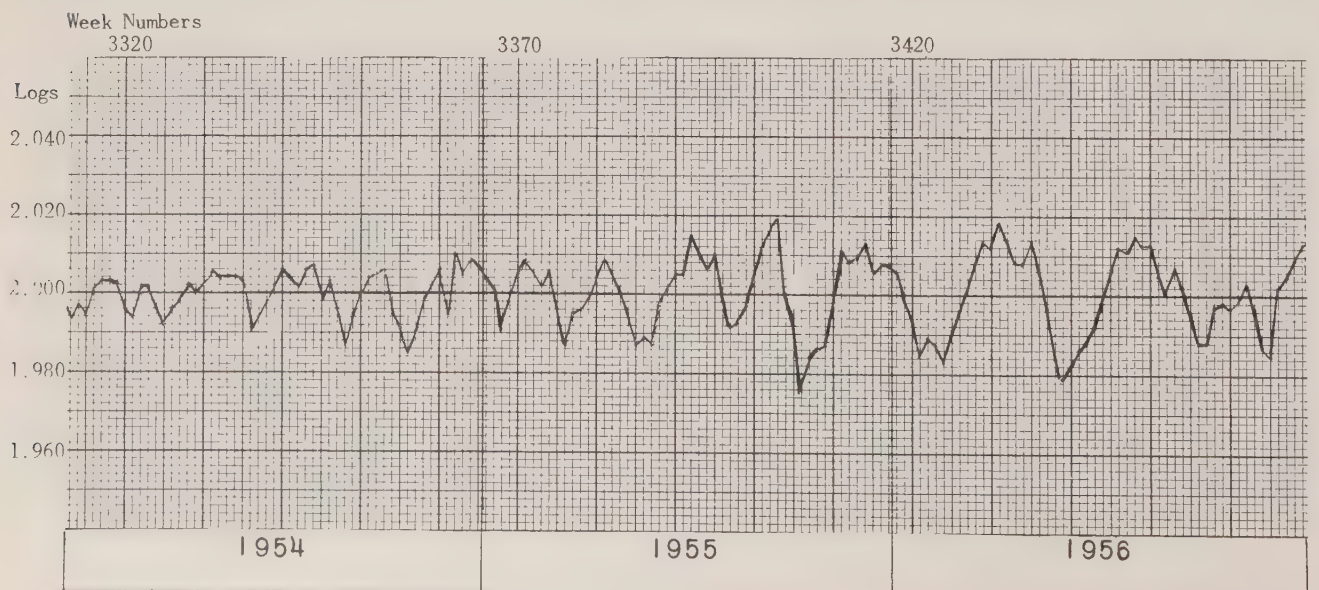
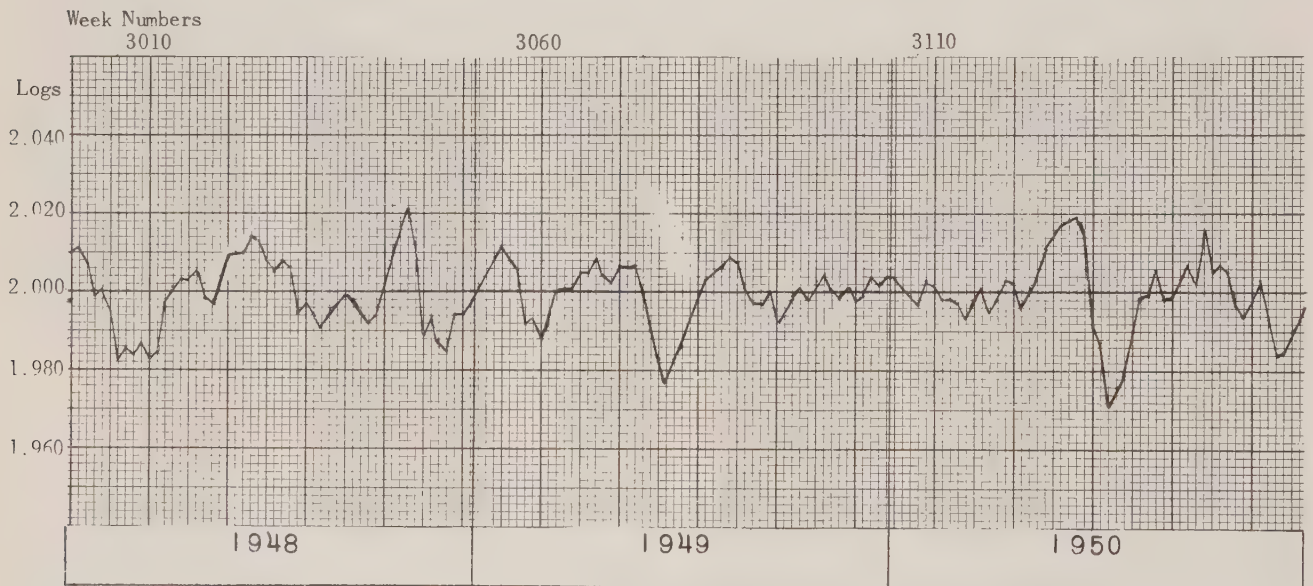
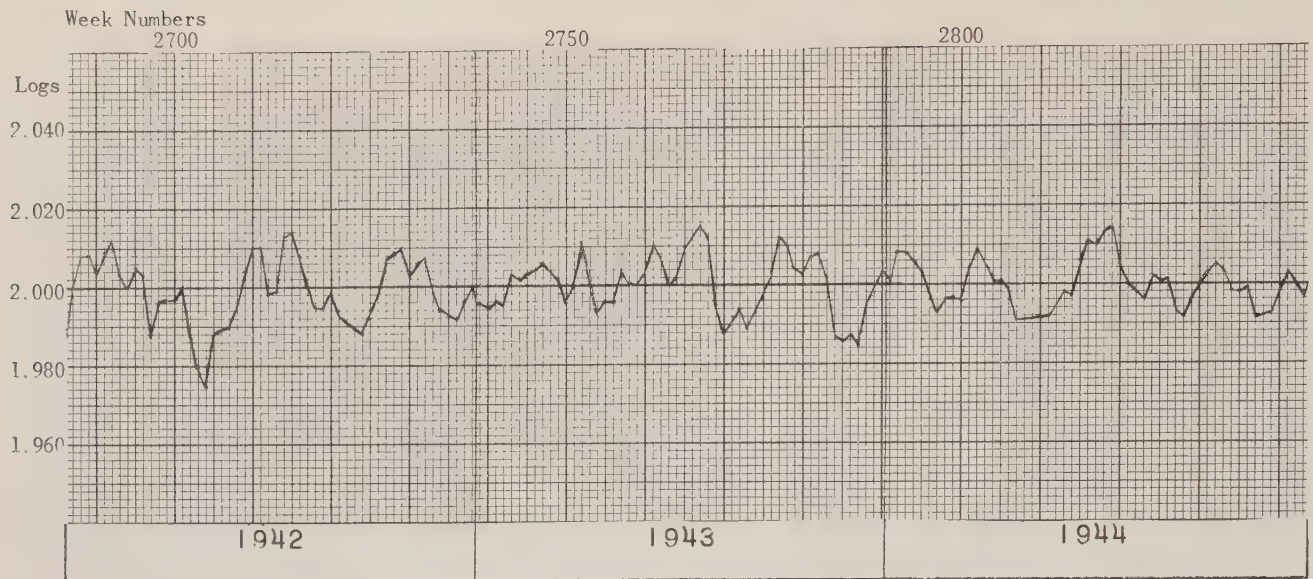


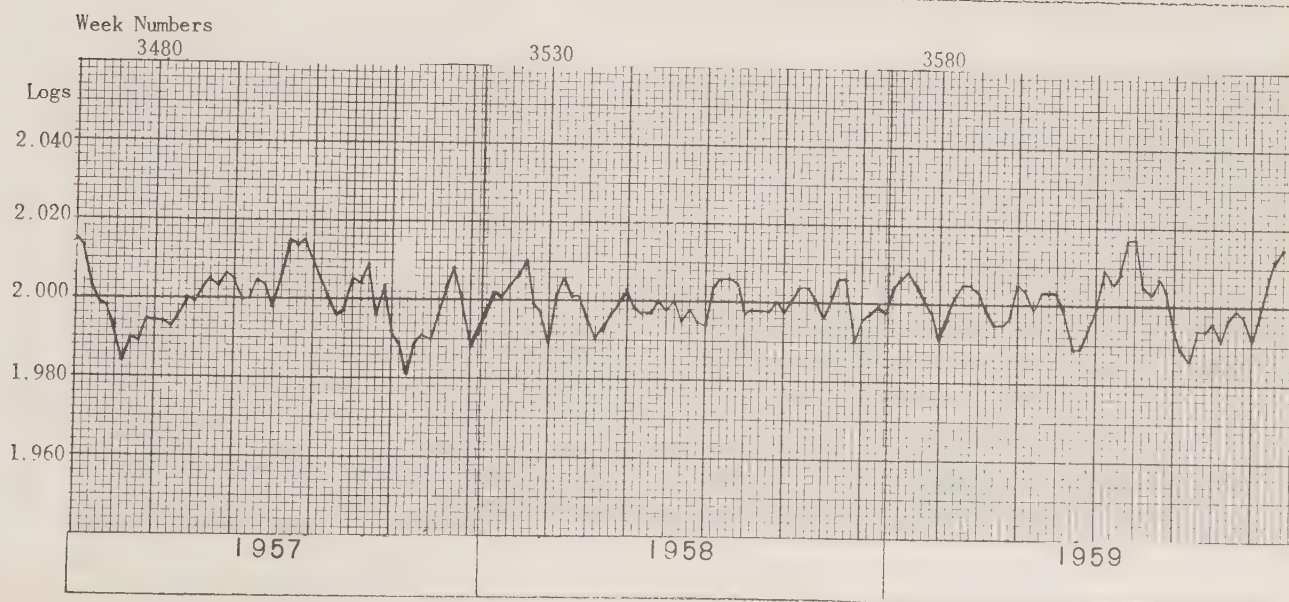
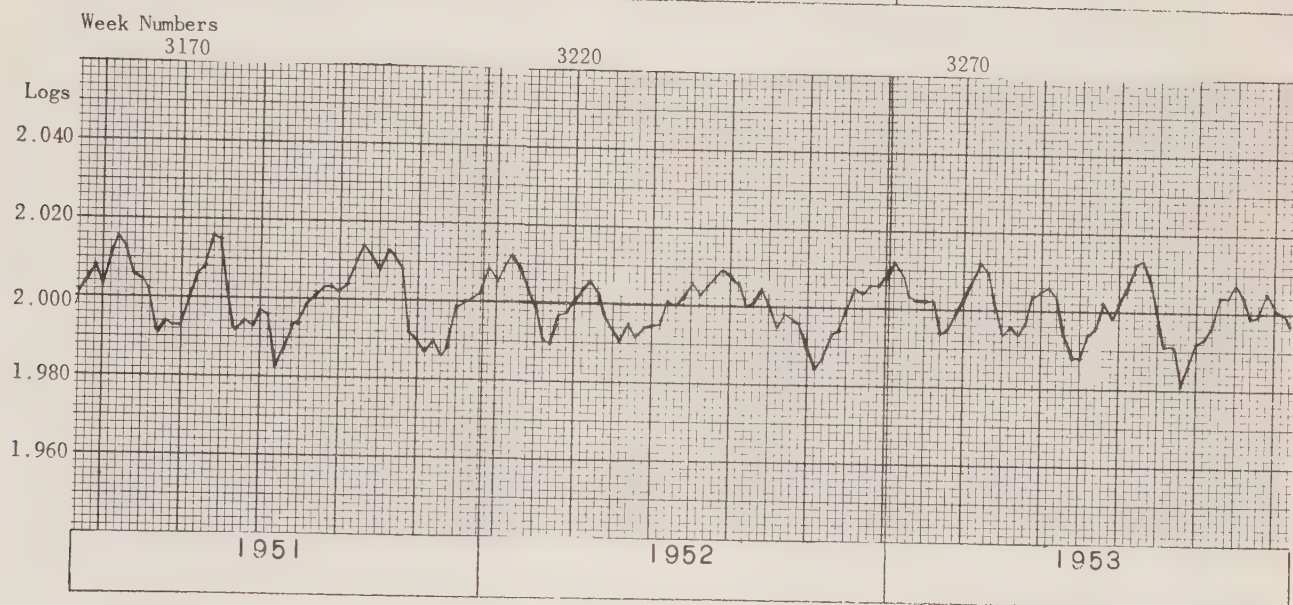
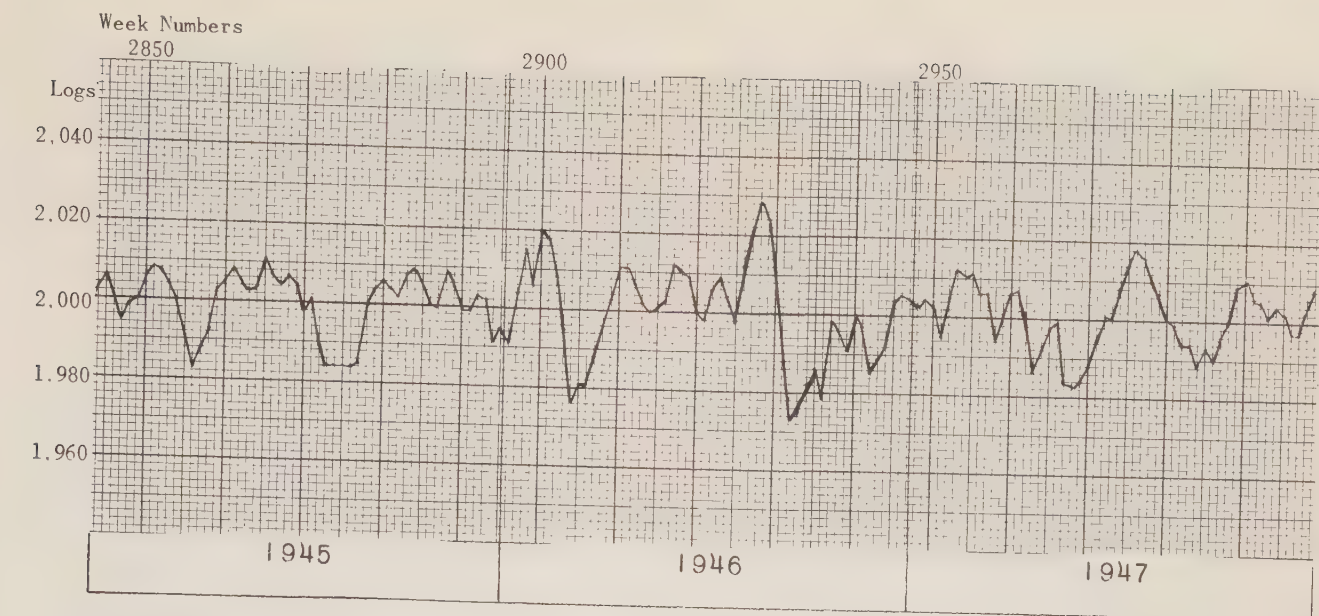












throw light on cause.

This paper will describe the data used, describe the method of analysis, present the results, and venture into some discussion of possible cause or causes.

THE DATA

For data we used weekly averages computed from daily closings of Dow-Jones Industrials, correct to two decimals, rounded, from January 9, 1897 through February 13, 1960 (the *Dow-Jones Averages*, *Barrons*, & newspapers). This period covers 63 years or 3,294 weeks. It is long enough to encompass 192 repetitions of a 17.16-week cycle.

Figures prior to July 31, 1914, at which time the exchange was closed because of World War I, are not comparable with figures after December 12, 1914 when the exchange reopened. To make them comparable the earlier figures were divided by 1.38.

Values between July 30, 1914 and December 12, 1914 were interpolated.

Data were converted to 3-place logs. The analysis was made with the logs.

THE ANALYSIS

To minimize the distorting effect of longer fluctuations and to permit the use of limited data, modified means, etc., in the periodic tables, the logs of the data were expressed as deviations from their 17-week arithmetic moving average. (A 17-week arithmetic moving average of the logs of the data is, of course, a 17-week geometric moving average of the data themselves.) These deviations are charted in Fig. 1.

This manipulation is akin to expressing data as deviations from a 12-month (or 52-week) moving average when studying a seasonal cyclic variation.

This manipulation introduces serial correlation, but this introduction is of no particular matter, as serial correlation already existed in the original data.

THE AVERAGE LENGTH OF THE CYCLE

Preliminary investigation of the so-called 17-week cycle in stock prices suggested a cycle length of 17.12 weeks for the period 1928 to 1957 (Dewey, 1958). The data used in the preliminary analysis were those of the Standard and Poor's Corporation Industrial Stock Price Index.

The deviations (expressed for convenience as values above or below 2.000) were, therefore, posted into a 17.12-week periodic table based on the week ending February 27, 1897 and every 17.12 weeks forward from that time. The

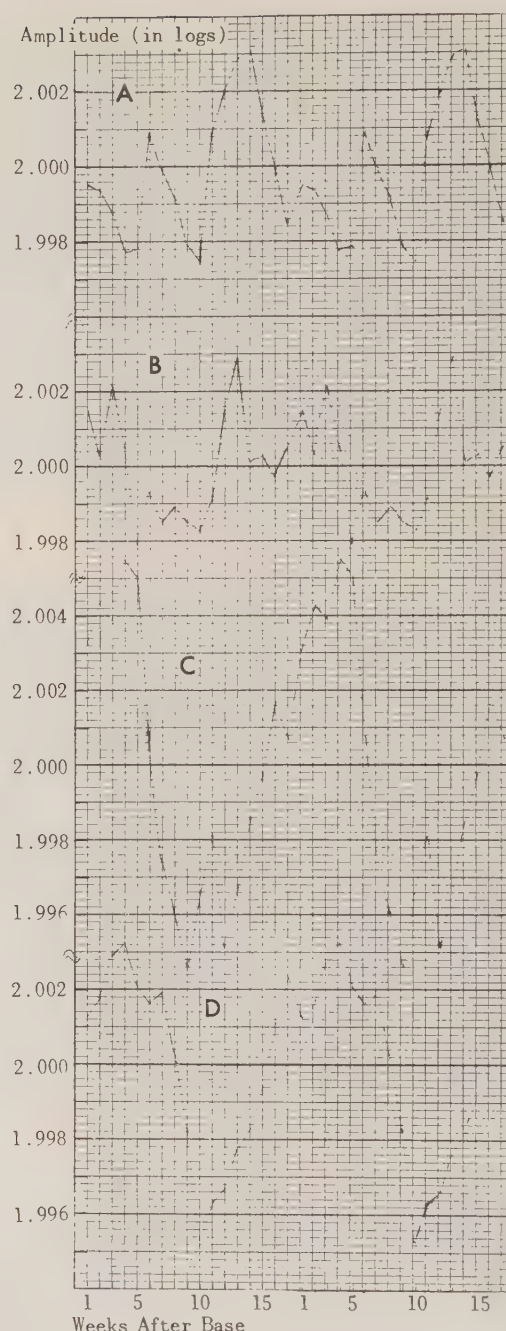


FIG. 2: THE 17.12-WEEK PERIODIC TABLE

Averages of Each Quarter of the 17.12-Week Periodic Table of the Logs of the Data Ex Their 17-Week Moving Average Trend. (From lines A, B, C, and D of Table 1 on page 100.)

The average wave is repeated by means of a broken line.

The fact that successive groups crest and trough progressively to the right suggests a length slightly longer than 17.12 weeks.

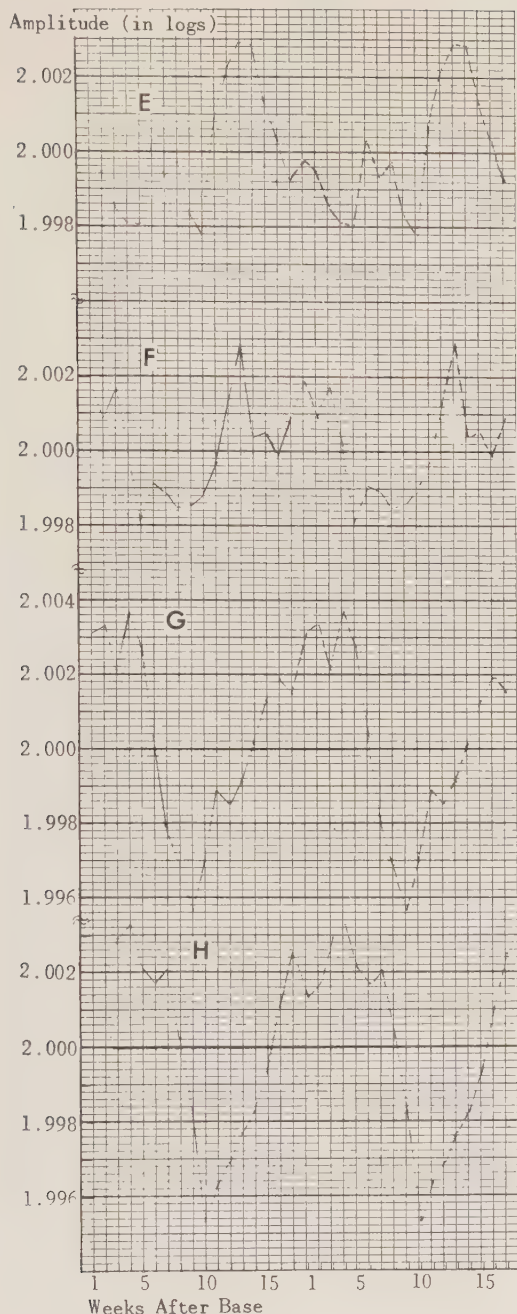


FIG. 3: THE 17.12-WEEK TABLE,
MODIFIED

Averages of Each Quarter of the 17.12-Week Periodic Table of the Logs of the Data Ex Their 17-Week Moving Average Trend, All Values Limited to ± 20 . (From lines E, F, G, and H of Table 1 on page 100.)

The average wave is repeated by means of a broken line.

By limiting the values to ± 20 some of the distortions in Fig. 2 due to randoms are eliminated.

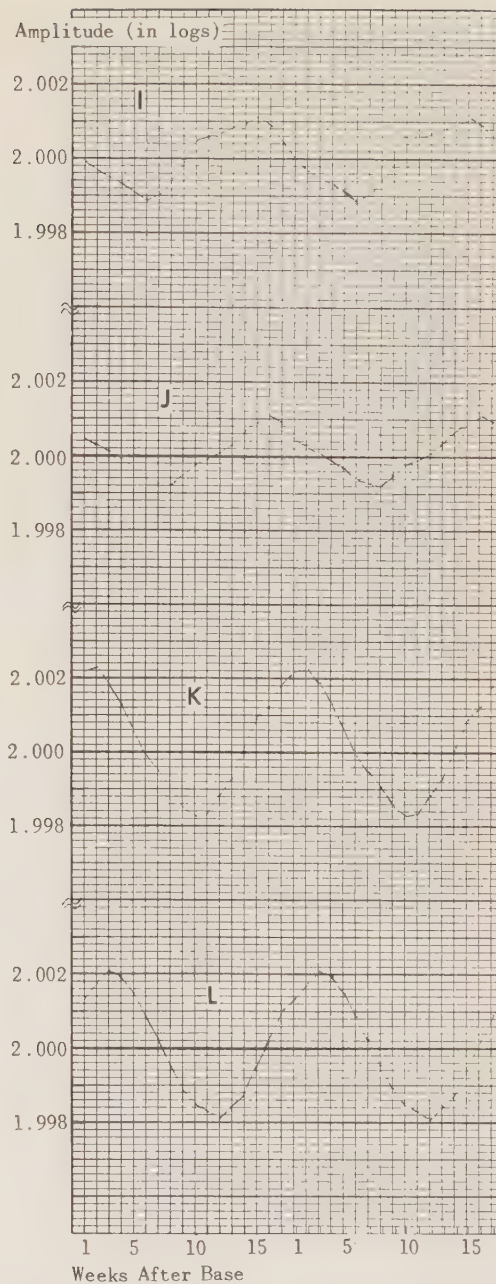


FIG. 4: THE 17.12-WEEK TABLE,
MODIFIED AND SMOOTHED

Averages of Each Quarter of the 17.12-Week Periodic Table of the Logs of the Data Ex Their 17-Week Moving Average Trend, All Values Limited to ± 20 and Smoothed by a 9-Week Moving Average. (From lines I, J, K, and L of Table 1 on page 100.)

The average wave is repeated by means of a broken line.

The smoothing process further minimizes randoms and permits the slippage to be measured more accurately. The slippage proves to be 6.6 positions in 138 lines or approximately .0478 weeks per cycle.

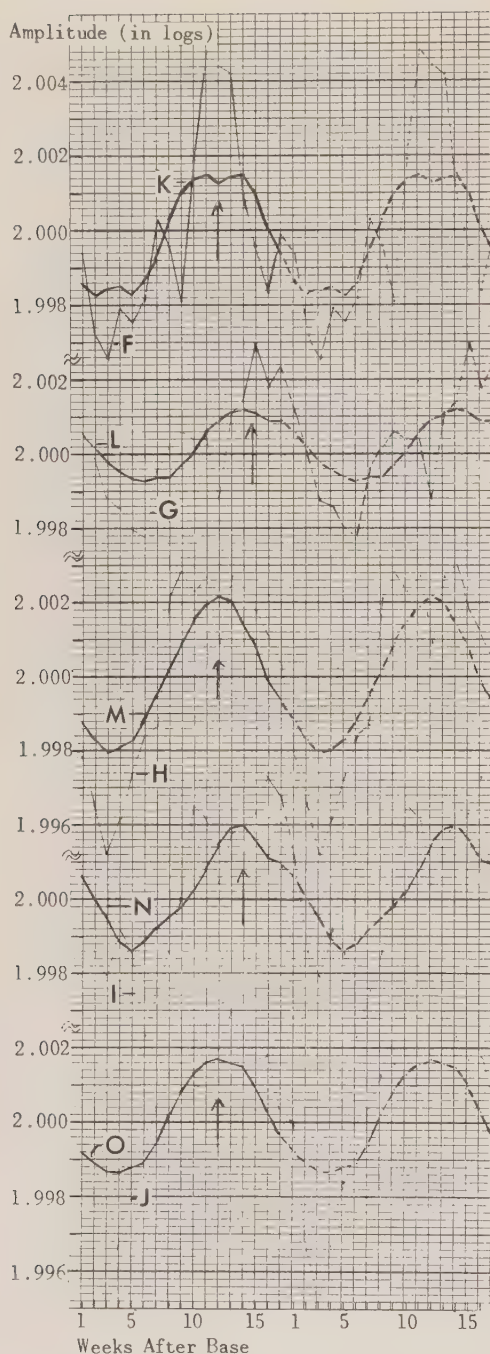


FIG. 5: THE 17.166-WEEK PERIODIC TABLE.

Light Lines:

Averages of Each Fifth of the 17.166-Week Periodic Table of the Logs of the Data Ex Their 17-Week Moving Average Trend, All Values Limited to ± 20 . (From lines F, G, H, I, and J of Table 2 on page 101.)

Heavy Lines:

The Light Line Values Smoothed by a 9-Week Moving Average. (From lines K, L, M, N, and O of Table 2.)

In all cases, the average wave is repeated by means of a broken line.

base is the week before the first deviation. Base numbers were rounded to the nearest integer. Values of .5 were rounded up. The table was arbitrarily averaged by quarters (46 line sections) with the results shown in Table 1, lines A, B, C, and D, and plotted in Fig. 2, curves A, B, C, and D, respectively. In the actual table, minus values were entered in red. This expedient gives immediate visual evidence of concurrent longer and shorter cycles and shows us how to average the sections so as to eliminate them (Dewey, 1951).

Excessive randoms can be minimized by limiting the deviations to $\pm .020$, as any values over these amounts are obviously due to randoms in excess of the average strength of $\pm .003$, unless the cycle is of varying length and/or of varying amplitude (Dewey, 1954). A reaveraging of the periodic table using data limited to $\pm .020$ is shown in Table 1, lines E, F, G, and H, and charted in Fig. 3, curves E, F, G, and H, respectively. It is clear that the cycle slips to the right from section to section at the rate of a little over two weeks per section (seven weeks in three blocks of 46 cycles each or 138 cycles). This slippage suggests a length about $7/138$ of a week (.05 weeks) longer than 17.12 weeks, or a length of 17.17 weeks 120.19 days.

We can refine this estimate by computing 9-week moving averages of lines E, F, G, and H with the results shown in Table 1, lines I, J, K, and L, and charted in Fig. 4, curves I, J, K, and L. This slippage can now be computed, first block to fourth block, as 6.6 weeks in 138 cycles or .0478 weeks per cycle. The indicated length is thus 17.1678 weeks (120.174 days). Averaging blocks one and two and blocks three and four, we find the slippage to be 4.425 weeks in 92 cycles or .0481 weeks per cycle. This method gives a length of 17.1681 weeks or 120.176 days. Either of these lengths will do for a first approximation.

For further refinement we now construct a new periodic table, for convenience 17.1666 weeks in length instead of the indicated 17.1681 weeks, as some further adjustment will doubtless be required anyway.

How shall we average this $17 \frac{1}{6}$ -year periodic table? A 5-section moving average of a 5-section moving average of our original periodic table shows bands of color moving across the table in such a way as to suggest that the greatest elimination of distorting cycles would be effected by averaging the new table by fifths—that is, by blocks of 36 cycles each.* This was done with the results shown in Table 2, lines A, B, C, D, and E. These results are not charted.

*Section moving averages are explained later in this paper.

With data limited to ± 0.020 we get the results shown in Table 2, lines F, G, H, I, and J, and charted in Fig. 5, lines F, G, H, I, and J, respectively.

Nine week moving averages of the five blocks are then computed with the results shown in Table 2, lines K, L, M, N, and O. These moving averages are plotted in Fig. 5, correspondingly lettered. It is clear that, on the whole, the waves slip neither to the right nor to the left. The length of 17.16 weeks is thus seen to be approximately correct.

For further refinement we now compute 6-week moving averages of the 9-week moving averages, equalize around the X axis, and plot the crossovers of each block of 36 cycles on an enlarged scale as in Fig. 6. This refinement times each 36-cycle section with greater accuracy.

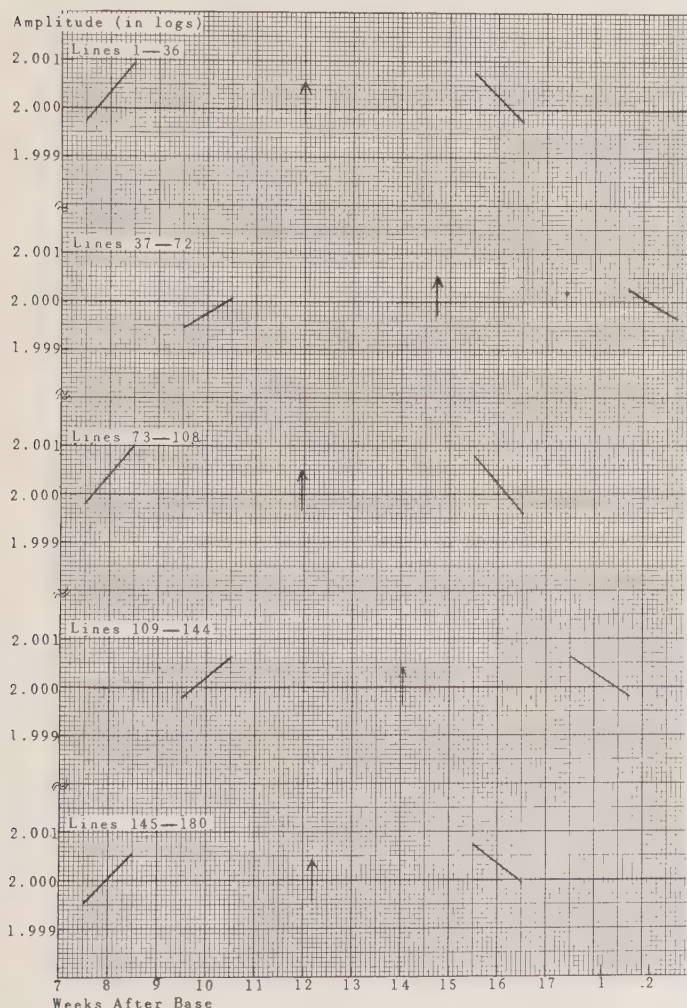


FIG. 6: THE 17.166-WEEK TABLE:

TIMING

Crossover Points and Idealized Timing of Crests for Each Fifth of the 17.166-Week Periodic Table

There seems to be no reason to change our initial assumption that the cycle is, on the average, about 17.16 weeks or 120.16 days long, plus or minus perhaps .015 week or .1 day.

SHAPE AND TIMING OF THE CYCLE

Now that we have the length of the cycle, let us average the entire 17 1/6-week periodic table, with data limited to ± 0.020 in order to get the average shape and timing of the waves. This has been done with the results shown in Table 2, line P. The curve is charted in Fig. 7. Lows come at position 3; highs at position 13. These positions correspond to week number 350 (week ending March 20, 1897) for a low, week number 360 (week ending May 29, 1897) for a high. Other lows and highs come, ideally, at 17 1/6 weeks forward from this time. This method of reckoning puts the latest ideal 1959 low at week number 3,612, or week ending September 26, 1959; the latest ideal 1959 high at week number 3,621 or week ending November 28, 1959.

THE STRENGTH OF THE CYCLE

The strength is seen to average 1.9967 at ideal time of low; 2.0026 at ideal time of high. These values are in logs. Converting to antilogs we see that, on the average, the waves run from 99.24 to 100.60 or an over-all average move of 1.36% of trend in 8 1/2 weeks or 2.72% of trend, round trip in 17 1/6 weeks; this amounts to 8.16% of trend per year.

IRREGULARITIES

When we study Fig. 1, it is clear that our cycle comes by fits and starts. Sometimes it

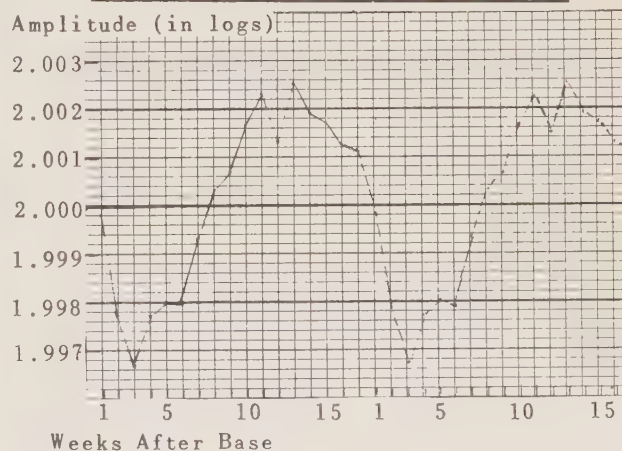


FIG. 7: THE 17.166-WEEK TABLE:

SUMMARY

Averages of All Five Fifths (180 Lines) of the 17.166-Week Periodic Table. (From line P of Table 2 on page 101.)

TABLE 1: THE 17.12-WEEK PERIODIC TABLE

Averages of Each Quarter of the 17.12-Week Periodic Table of the Logs of the Data* Ex Their 17-Week Moving Average Trend

*Data used throughout the work are Dow-Jones Industrial Stock Price Averages.

The deviations thus obtained are expressed as values above and below 2.000, and multiplied by 1,000 to put the decimal in a more convenient location.

LINE:	WEEKS AFTER BASE																
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
A	-0.46	-0.61	-1.24	-2.22	-2.17	0.84	-0.02	-0.83	-2.13	-2.57	0.91	2.00	2.98	3.09	1.28	-0.02	-1.46
B	1.50	0.30	2.17	0.48	-2.07	-0.67	-1.46	-1.11	-1.48	-1.74	-0.87	1.52	2.83	0.15	0.28	-0.24	0.18
C	3.22	4.33	3.93	5.46	5.07	0.80	-2.52	-3.57	-5.34	-3.41	-1.29	-4.83	-3.26	-1.41	-0.24	1.67	0.76
D	1.26	1.76	2.93	3.22	2.07	1.63	1.96	0.24	-1.76	-4.67	-3.61	-3.37	-2.34	-1.70	-0.72	1.07	2.39
E	-0.20	-0.54	-1.41	-1.85	-1.89	0.33	-0.61	-0.26	-1.72	-2.20	0.85	2.22	2.29	2.87	1.33	0.35	-0.76
F	1.83	0.85	1.72	0.00	-1.91	-0.89	-1.13	-1.48	-1.41	-1.15	-0.33	1.28	2.23	0.39	0.54	-0.13	0.89
G	3.09	3.37	2.09	3.67	2.78	0.55	-2.09	-3.13	-4.37	-3.00	-1.15	-1.43	-0.85	0.17	1.26	1.89	1.50
H	1.37	1.76	2.93	3.26	2.07	1.72	2.02	0.22	-1.76	-4.65	-3.61	-3.41	-2.33	-1.76	-0.72	1.07	2.54
I	-0.07	-0.35	-0.56	-0.63	-0.91	-1.13	-0.97	-0.57	-0.04	0.42	0.60	0.70	0.82	0.98	1.17	0.92	0.47
J	0.46	0.32	0.13	-0.02	-0.27	-0.60	-0.73	-0.78	-0.47	-0.21	-0.05	0.06	0.32	0.68	0.91	1.13	0.99
K	2.20	2.22	1.85	1.29	0.64	-0.04	-0.54	-0.93	-1.43	-1.72	-1.62	-1.18	-0.67	0.17	0.87	1.23	1.80
L	1.39	1.78	2.08	1.99	1.51	-0.84	0.34	-0.43	-1.05	-1.47	-1.74	-1.85	-1.59	-1.24	-0.53	0.20	0.90

KEY

SECTION 1: Raw Deviations (Charted in Fig. 2)

Line A: Average—1st 46 lines of the periodic table.

Line B: 2nd 46

Line C: 3rd 46

Line D: 4th 46

Line G: 3rd 46

Line H: 4th 46

SECTION 3: Nine-Week Moving Average of the Values in Section 2 (Charted in Fig. 4)

Line I: Average—1st 46 lines of the periodic table, limited and smoothed.

Line J: 2nd 46

Line K: 3rd 46

Line L: 4th 46

SECTION 2: Deviations Limited to 20 (Charted in Fig. 3)

Line E: Average—1st 46 lines of the periodic table, limited.

Line F: 2nd 46

TABLE 2: THE 17.166-WEEK PERIODIC TABLE

Averages of Each Fifth of the 17.166-Week Periodic Table of the Logs of the Data Ex Their 17-Week Moving Average Trend

The deviations thus obtained are expressed as values above and below 2.000, and multiplied by 1,000 to put the decimal in a more convenient location.

LINE	WEEKS AFTER BASE																
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
A	-0.34	-2.97	-4.08	-2.56	-2.14	-1.19	0.31	-0.97	-1.64	1.28	4.47	4.92	4.58	1.08	-1.11	-2.25	-0.81
B	1.22	-0.31	-0.97	-0.92	-1.53	-2.69	-0.47	0.22	0.50	0.19	0.39	-1.53	-0.22	1.00	2.36	1.69	2.94
C	-2.47	-4.97	-6.42	-5.22	-3.89	-3.19	-4.31	0.92	2.31	1.06	2.56	0.92	3.06	3.78	4.06	3.92	3.53
D	1.72	-1.08	-1.64	0.06	-1.11	-4.97	-5.17	-4.00	-1.42	3.19	2.00	-0.64	1.53	1.33	1.67	3.33	3.17
E	-0.06	-3.44	-4.94	-3.50	-1.72	-0.52	0.47	1.28	1.64	0.83	0.86	2.03	2.97	2.25	1.69	1.64	0.28
F	-0.61	-2.81	-3.42	-2.11	-2.42	-1.81	0.33	-0.34	-1.86	1.94	4.75	4.44	4.22	1.03	-0.36	-1.56	-0.06
G	1.06	-0.11	-1.19	-1.39	-1.97	-2.17	-0.42	0.19	0.56	0.34	0.34	-1.22	1.06	1.44	2.97	1.78	2.39
H	-2.17	-3.53	-4.83	-3.75	-2.61	-1.58	-1.25	2.02	2.13	2.31	2.22	0.92	2.69	3.00	1.75	1.08	0.42
I	0.89	-1.42	-2.14	-0.83	-1.44	-3.72	-2.36	-1.34	0.00	2.53	2.19	0.42	1.83	1.97	2.31	3.28	2.81
J	-0.08	-3.44	-4.83	-3.25	-1.64	-0.78	0.36	1.28	1.64	1.03	1.00	2.03	2.97	2.31	1.69	1.75	0.36
K	-1.4	-1.7	-1.6	-1.5	-1.7	-1.4	-0.6	0.3	1.0	1.4	1.5	1.3	1.4	1.5	1.0	0.1	-0.6
L	0.6	0.2	-0.2	-0.4	-0.6	-0.7	-0.6	-0.6	-0.3	0.0	0.6	0.9	1.1	1.2	1.1	0.9	0.9
M	-1.2	-1.7	-2.0	-1.9	-1.7	-1.2	-0.5	0.2	0.9	1.5	1.9	2.2	2.0	1.4	0.8	-0.1	-0.6
N	0.6	0.0	-0.5	-1.1	-1.4	-1.2	-0.8	-0.5	-0.2	0.2	0.8	1.5	1.9	2.0	1.6	1.1	1.0
O	-0.8	-1.1	-1.3	-1.3	-1.2	-1.1	-0.6	0.2	0.9	1.3	1.6	1.7	1.6	1.5	1.0	0.3	-0.3
P	-0.18	-2.24	-3.26	-2.28	-2.00	-2.02	-0.68	0.36	0.62	1.62	2.22	1.30	2.56	1.94	1.68	1.28	1.18

KEY

SECTION 1: Raw Deviations (Not Charted)

Line A: Average—1st 36 lines of the periodic table.

Line B: 2nd 36

Line C: 3rd 36

Line D: 4th 36

Line E: 5th 36

Line J: 5th 36

SECTION 3: Nine-Week Moving Average of the Values in Section 2 (Charted in Fig. 5 by heavy lines)

Line K: Average—1st 36 lines of the periodic table, limited and smoothed.

Line L: 2nd 36

Line M: 3rd 36

Line N: 4th 36

Line O: 5th 36

SECTION 2: Deviations Limited to ± 20 (Charted in Fig. 5 by light lines)

by Line F: Average—1st 36 lines of the periodic table, limited.

Line G: 2nd 36

Line H: 3rd 36

Line I: 4th 36

SECTION 4: Average of Entire Table (Charted in Fig. 7)

Line P: Average—all five-fifths (180 lines) of the 17.166-week periodic table. (Data limited to ± 20 .)

Fig. 8: DOW-JONES INDUSTRIAL STOCK PRICE DEVIATIONS

AS CHARTED IN FIG. 1

FILTERED AND SMOOTHED

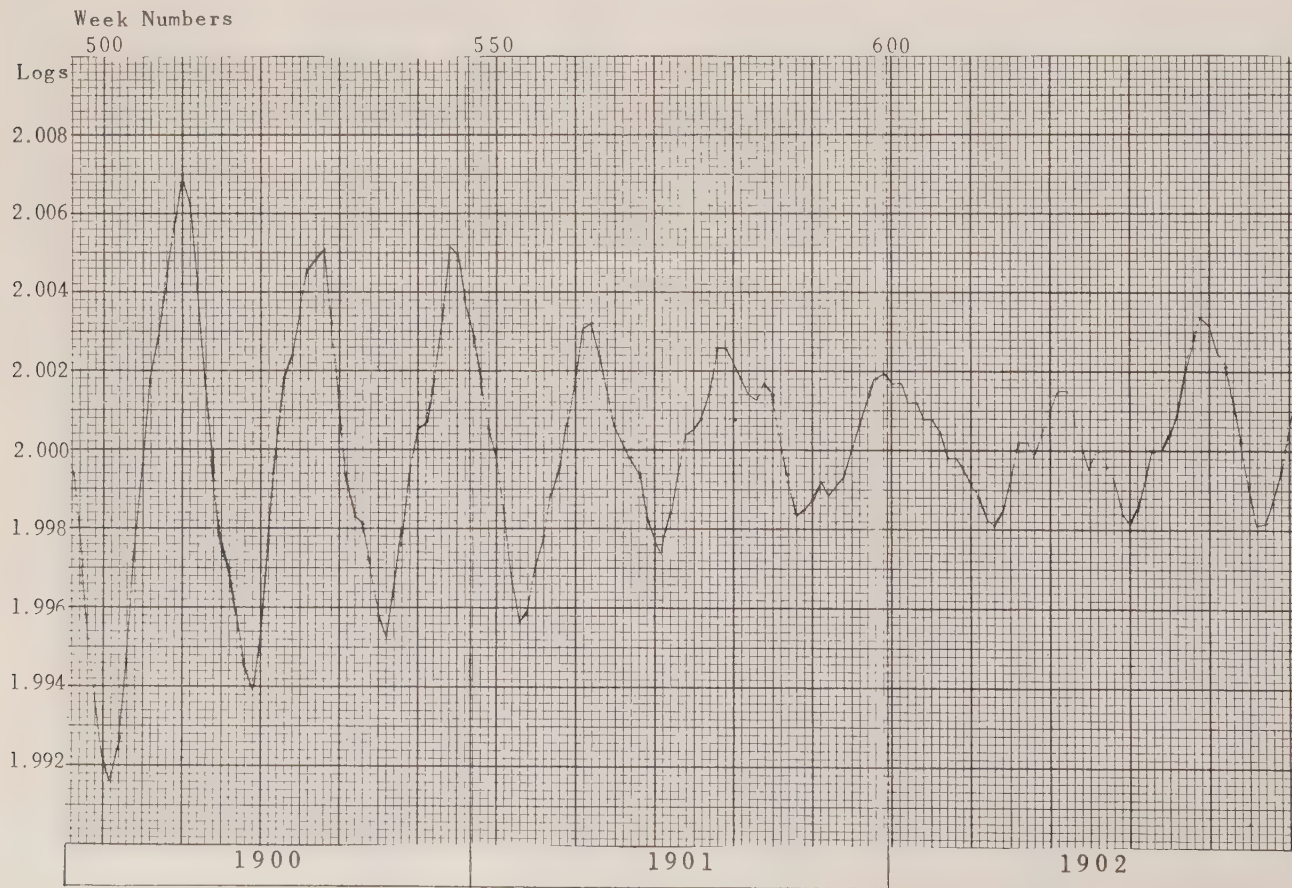
The filtering process, described in the text, has no effect upon any perfectly regular 17 1/6-week cycle present in the deviations. It has no effect upon the length of cycles of nearby periods, but does reduce their amplitude somewhat. Thus 15-, 16-, 18-, and 19-week cycles, if present, would be clearly visible. The filtering process minimizes and/or completely eliminates longer and shorter cycles. Consequently, it permits the 17 1/6-week cycle to vary in length and strength instead of forcing it into a rigid pattern.

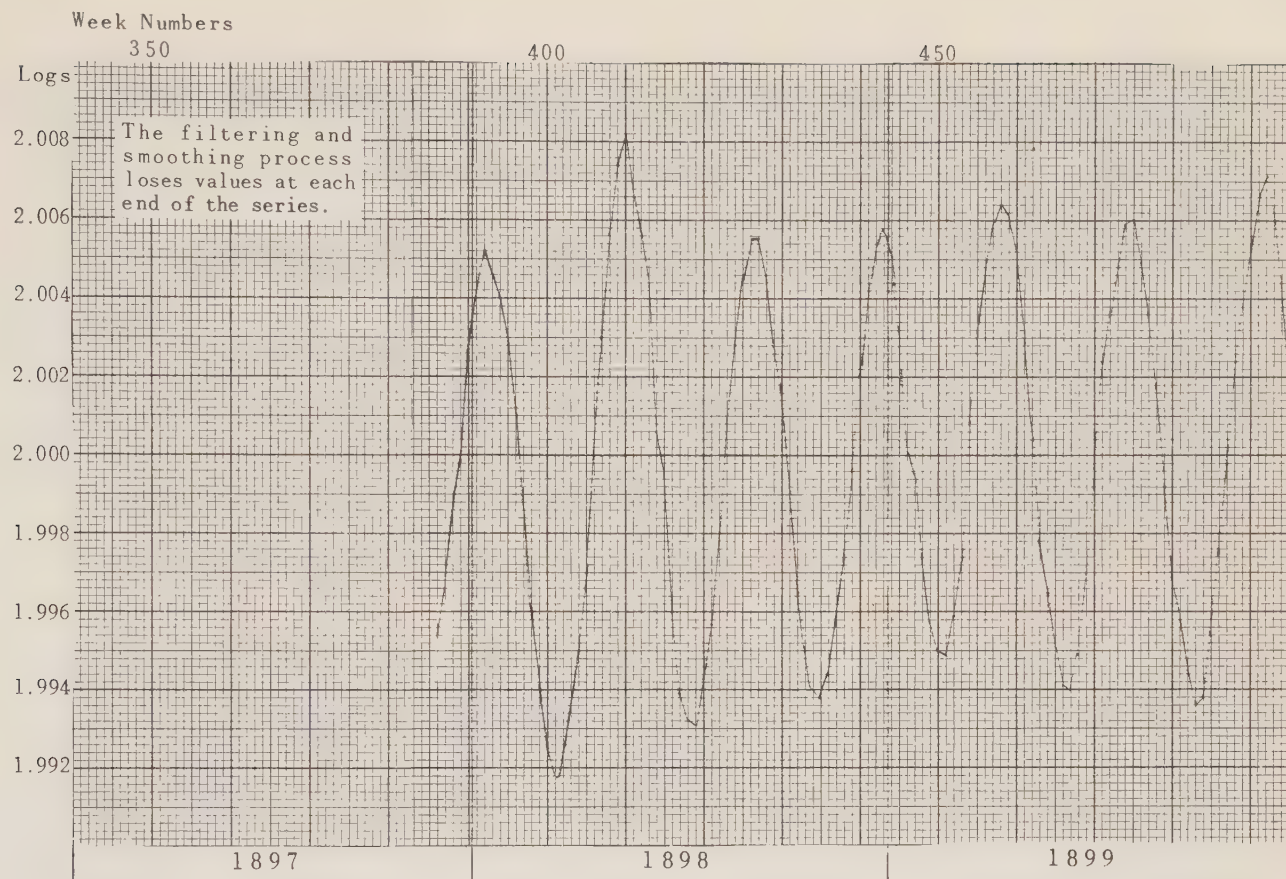
The filtering process minimizes randoms by reducing them by 1/5 and spreading the other 4/5 at 17-week intervals. This

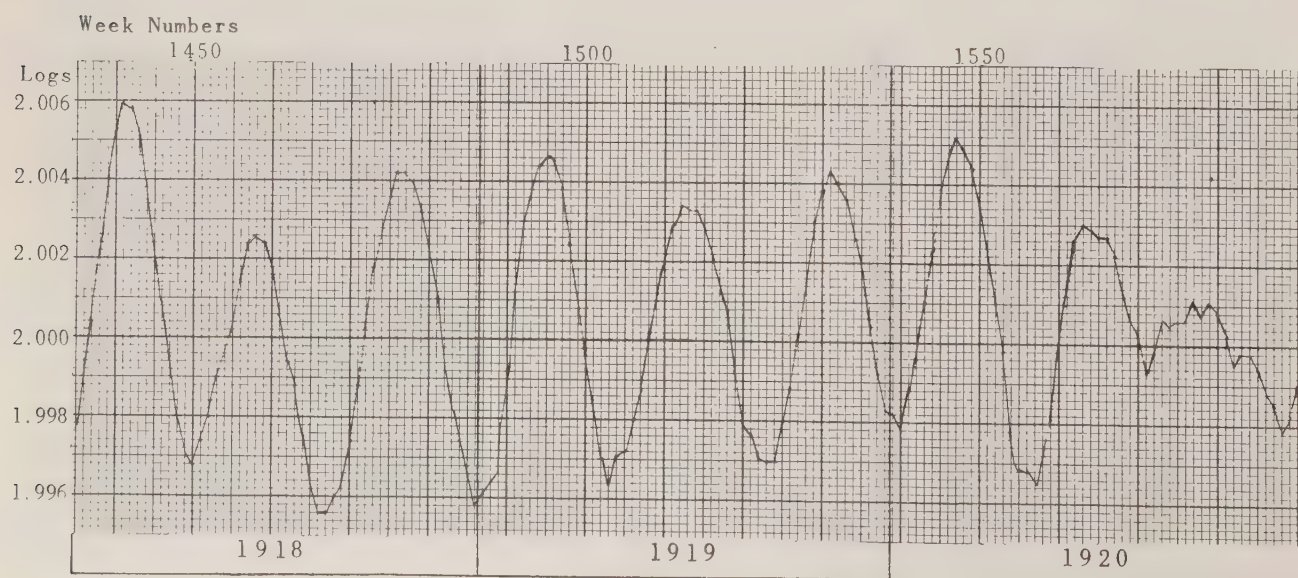
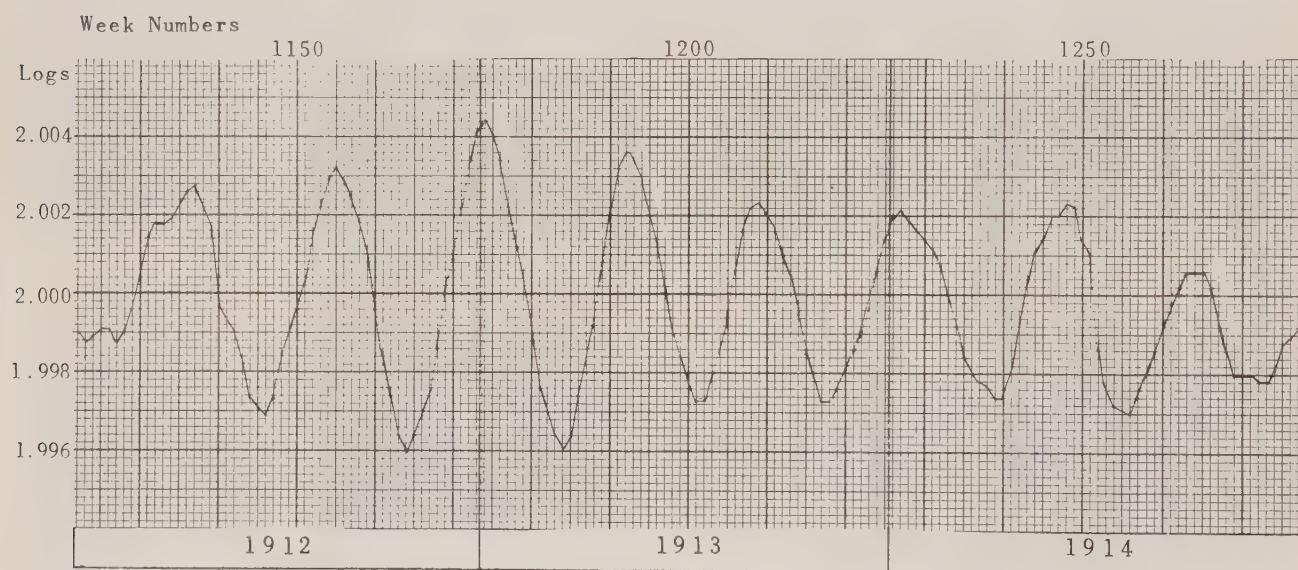
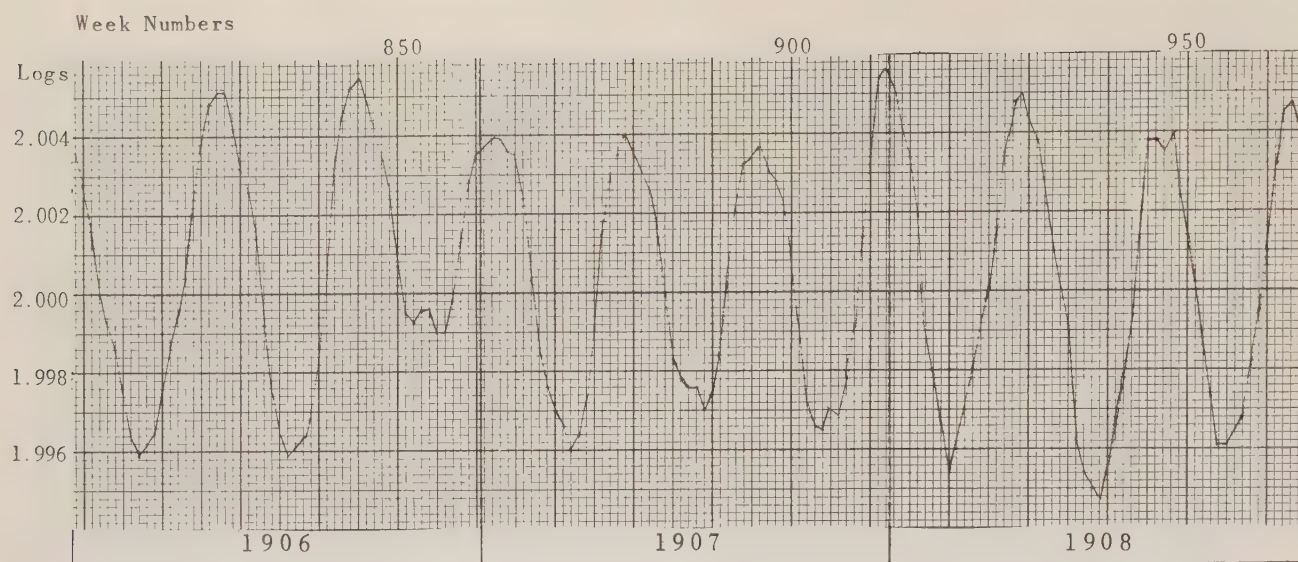
circumstance creates artifacts, but they are easily recognized as, in no case, can they persist for more than five waves. Successive artifacts are not in phase, except by accident.

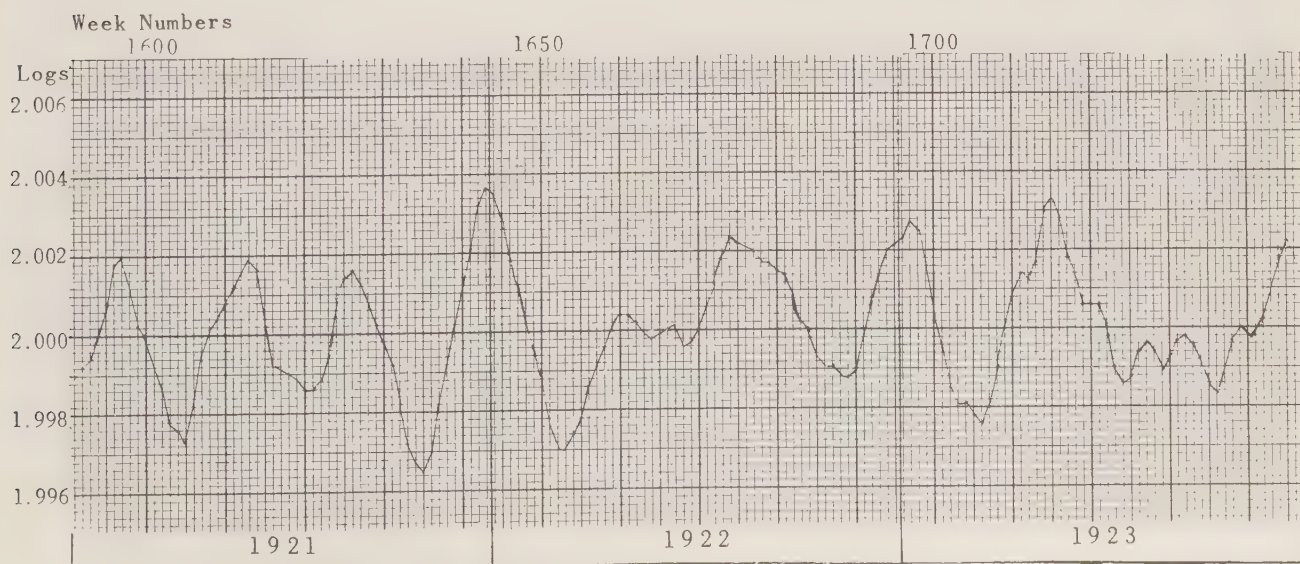
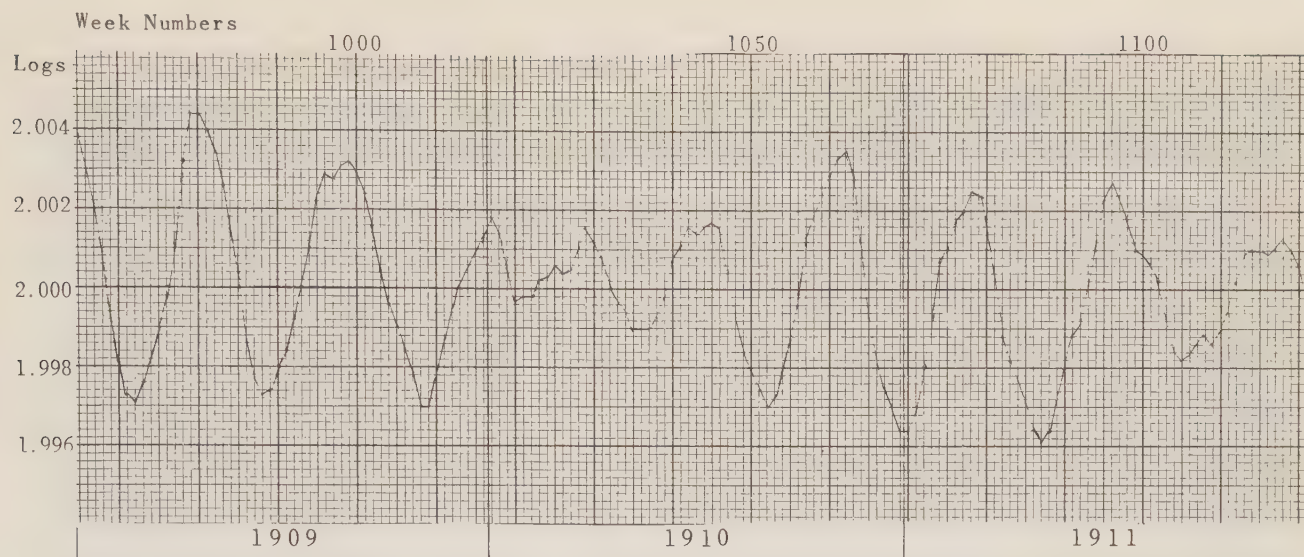
The smoothing process further minimizes randoms by distributing them over adjacent weeks. The smoothing process reduces the true strength of the cycle by about one half.

To offset the reduced amplitude due to the smoothing process, and to enable us to study variations of timing and amplitude, the vertical scale of Fig. 8 has been increased tenfold relative to the vertical scale of Fig. 1.

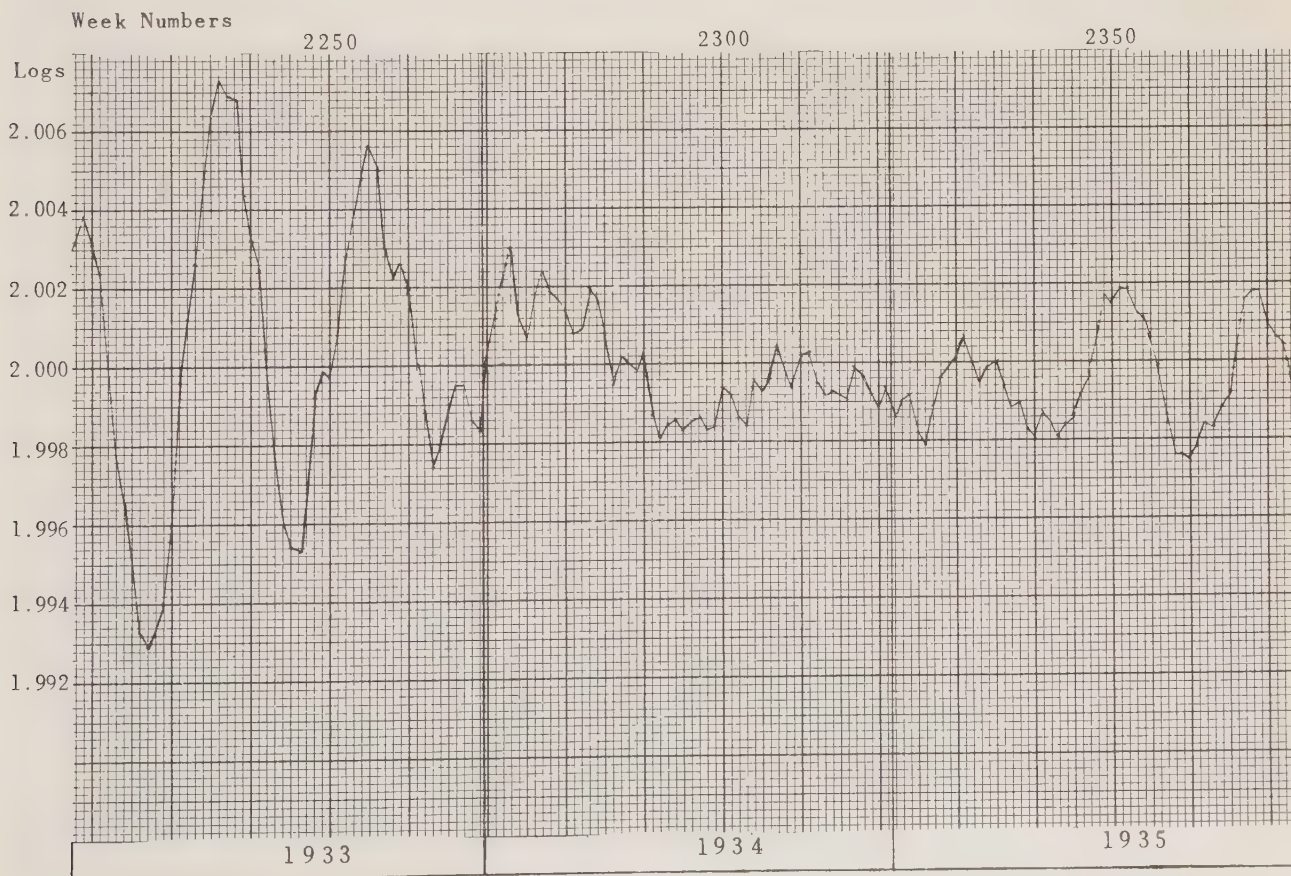
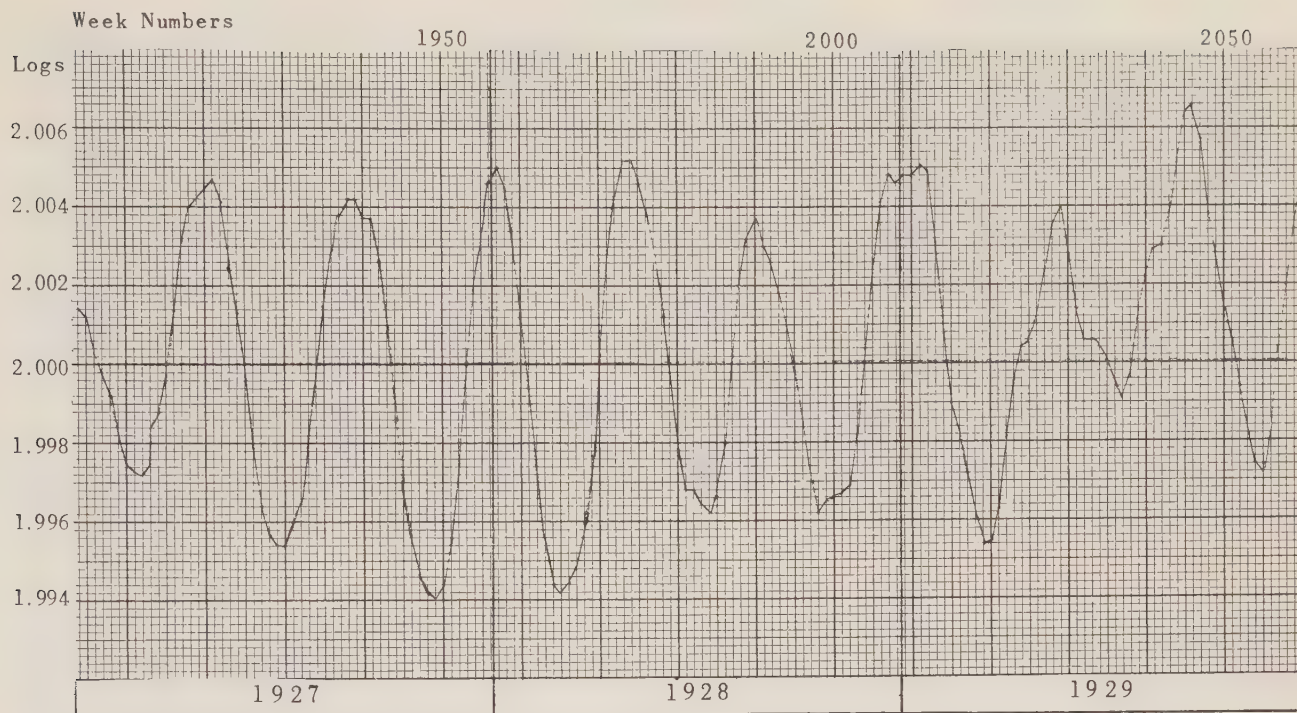


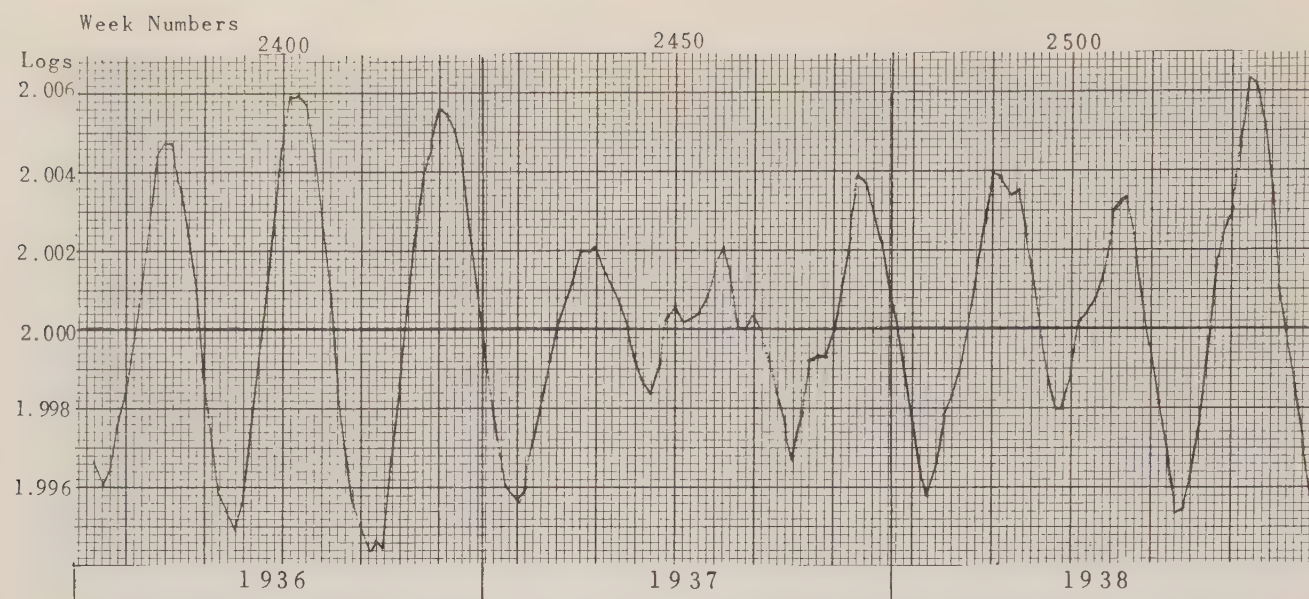


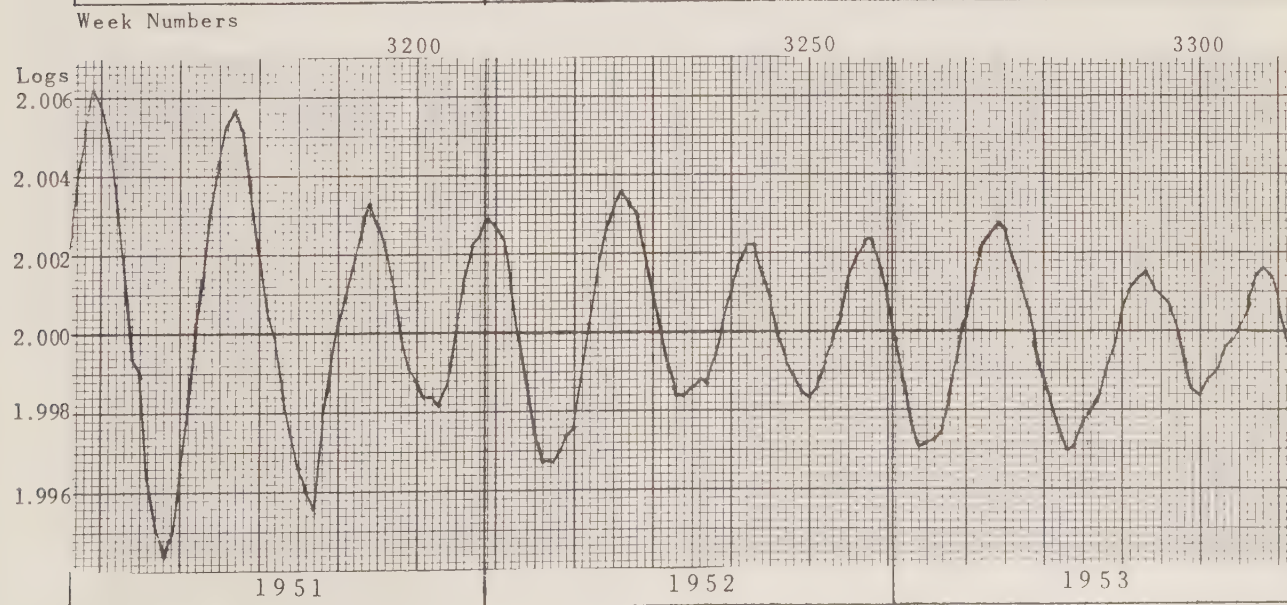
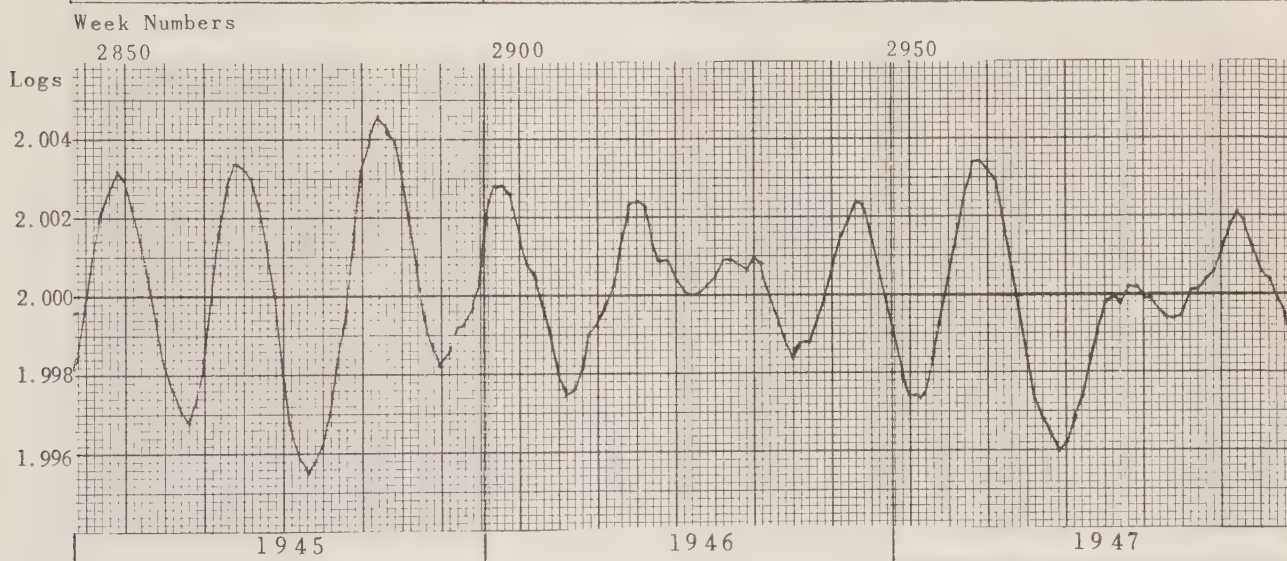
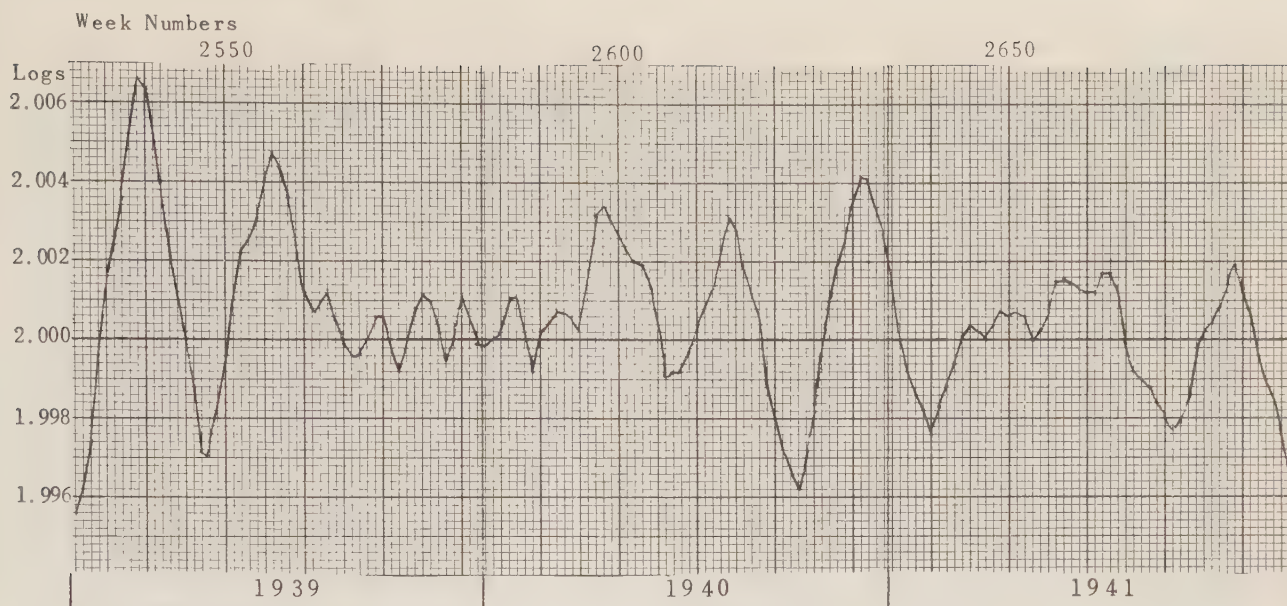


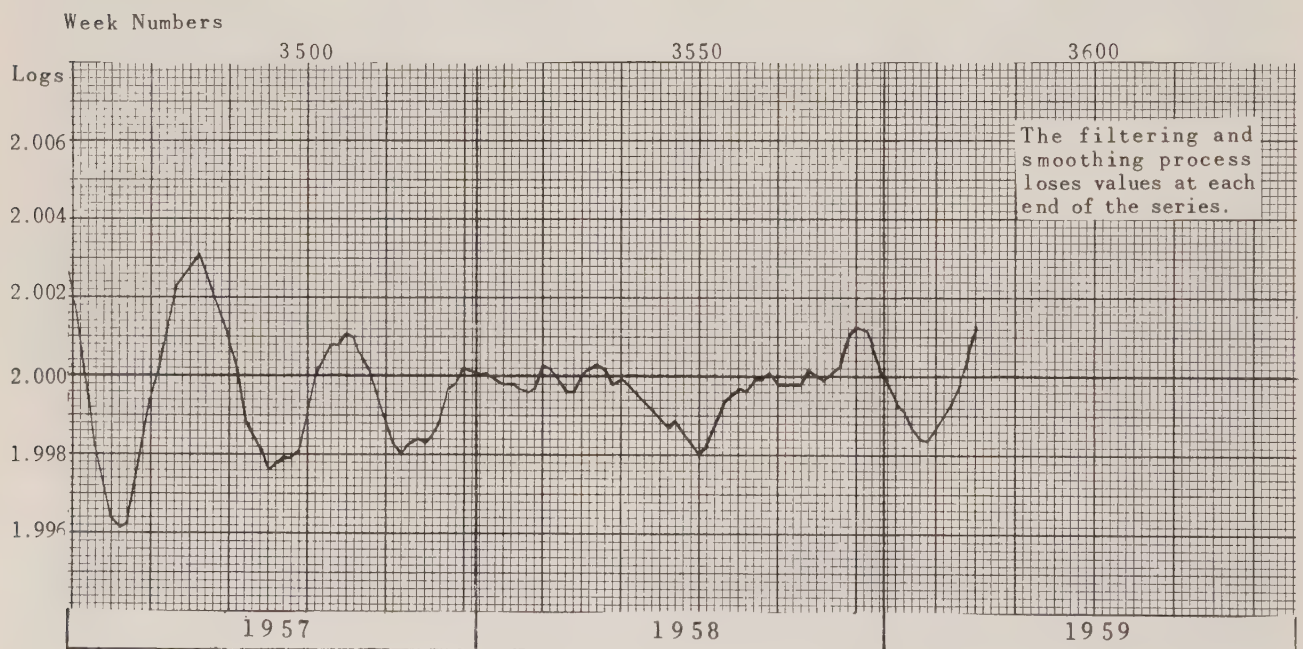












cycle to vary in *amplitude* and *length* and *phase*. It also has the advantage that it minimizes longer and shorter cycles. Randoms, too, are reduced to 1/5. The other 4/5 of any random are distributed at 17-week intervals, thus introducing a specious 17-week cycle, but such cycles are easily identifiable and persist for no more than five repetitions at the most (Dewey, 1956).

Still further to simplify the picture these 5-section moving average values are smoothed by a 9-week moving average. The final results are charted in Fig. 8. The minimizing of randoms and other cycles clarifies the behavior but it is still clear that the 17-week waves come and go, are strong and weak, come early, come late. Our original concept seems to be correct.

VARIATION IN WAVE LENGTH AND IN WAVE STRENGTH

To see if there is any regularity to these irregularities, or whether they are completely random, further study is required. Such study will be the subject of another paper.

CAUSE

Casual study of Fig. 1 suggests dynamic behavior which often takes about 1/3 of a year (17 weeks) to complete a wave.

Dynamic behavior would operate as follows: Let us imagine that something disturbs the market, let us say downward. The force of this something takes about eight or nine weeks to spend itself. Then a reaction sets in. This reaction takes another eight or nine weeks to spend itself. This makes a total of 16, 17, or 18 weeks to a wave. One wave then generates another and so on until the disturbance dies out. Then, at a later date, some other totally unrelated event starts another sequence of waves. And so on, at intervals, over the entire 63-year period.

This may indeed be the true explanation, but, if so, why would the generating events come at multiples of a uniform (17 1/6-week) interval? The dynamic theory would do very well to explain behavior of this sort over a 5- or 6-year period, but is hardly adequate to explain events that stay pretty much in phase—at least on the average—over more than half a century.

On the other hand, the behavior of the waves even when they are filtered and smoothed, as in Fig. 8, does not seem to be of the sort that we would get from some *single* perfectly regular cyclic force. For one thing, the variation from perfect timing is considerable; for another, the strength of the waves varies importantly. It is more or less as if we were dealing with the resultant of a welter of periodic forces, all about this same wave

1st block to 2nd block	17.24 weeks
2nd block to 3rd block	17.09 weeks
3rd block to 4th block	17.22 weeks
4th block to 5th block	17.11 weeks
Average	17.165 weeks
or	120.155 days

The clue provided by the foregoing table suggests the desirability of more intensive study of the variations of wave length (timing) and strength. To make such a study we must simplify the picture by eliminating extraneous cycles and randoms, insofar as this is possible.

Let us start by making a 5-section moving average of our 17 1/6-week periodic table. Such a section average is made by computing a 5-item centered moving average of each *column* of the table, all values limited to $\pm .20$ and interpolating a value every 103 weeks to make up the extra week present at this interval. (The table is 17 1/6 instead of merely 17 weeks long.) See the Appendix for the arithmetic.

[illegible]

Such a moving average has *no* effect whatever upon any perfectly regular 17-week cycle that might be present in the figures. It has the advantage that it permits an irregular 17-week

length, that sometimes reinforce, sometimes offset each other.

Nevertheless, the mean length of 17.166 weeks (120.166 days) does stand out conspicuously, and must be considered correct within probably 0.015 weeks or 0.10 days. This being so, we can look to see what else in nature, if anything, varies similarly.

SIDEREAL PERIODS OF THE PLANETS

Thirty-six times 120.16 days is 11.84 years. This length is very close to 11.86, the average sidereal period of Jupiter. In fact, it differs from the average sidereal period of Jupiter by less than successive revolutions of Jupiter (from perihelion to perihelion) differ from each other.

The theory has been advanced that the planets, in their revolution around the sun, generate electrical forces by cutting postulated lines of force originating in outer space. It is further theorized that these forces have terrestrial repercussions (Bousfield, 1948). If these theories should be true one would expect cycles equal in length to the sidereal periods of the planets, or simple fractions of such periods. Could our 120.16-day cycle be 1/36 of the sidereal period of Jupiter?

The answer is No. Our measurements are sufficiently exact so that we can be sure that there is no identity. One thirty-sixth of the sidereal period of Jupiter is 17.1928 weeks. This length is .0261 weeks (4.38 hours) longer than our observed length of 17.16 weeks. The difference is not great, but it amounts to .94 weeks from one block of 36 cycles to the next; 3.76 weeks from the 1st to the 5th block. If the stock price cycle were 1/36 of the sidereal period of Jupiter, the bottom curves of Fig. 5 would have been displaced 3 3/4 positions to the right.

There is no other planet whose sidereal period would seem to offer a reasonable correspondence.

SUNSPOTS

The matter of a possible 17.16-week cycle in sunspot numbers, or in sunspot numbers with alternate cycles reversed, has not, as far as I know yet been investigated.

VARIABLE STARS

There are a number of variable stars listed in the variable star catalog with periods of about 120 days (Kukarkin and Parenago, 1958). Only three of these periods have been measured with enough accuracy so that they can be disqualified from possible association with our 17.16-week (120.16-day) cycle. Any of the

others could be associated. Assuming a more or less random distribution of length from 119.5 to 120.5, one or more of them probably is. But this would have been true no matter what precise length our 17-week cycle had had. Therefore, the association, even if it were established, would mean nothing.

One curious fact does emerge from the scanning of the variable star catalog. If one lists all variable star lengths from say 115.50 days to 125.49 days, one finds an important concentration at 120 days, as shown in Table 3 and in Fig. 9. This concentration is too great to be easily the result of chance.

SYNODIC PERIODS

There is an association between planetary angles and terrestrial radio propagation quality (Nelson, 1951). If one terrestrial association has been discovered, there may be others. The subject is worthy of investigation, but unfortunately the information is not at hand with which to make the comparison. The average synodic period of each pair of planets is, of course, readily available, but as the orbits of the planets are eccentric, successive equal angular relationships will differ in time interval from year to year. For slow moving planets, the average length of a certain recurrence for the period 1897—1959 would not be the same as the average over the entire period of a revolution.

The matter of cause, then, must for the moment remain unknown. However, the precise wave length determination for the period under review is a first step in this direction. Even though it may not reveal the cause, it will serve to eliminate from consideration certain

TABLE 3

Number of Variable Stars
Listed in the *General Catalog of Variable Stars*,
First (1948) Edition
with Periods as Indicated

Period in Days	Number of Variable Stars
115.50—116.49	4
116.50—117.49	7
117.50—118.49	5
118.50—119.49	5
119.50—120.49	25
120.50—121.49	7
121.50—122.49	5
122.50—123.49	2
123.50—124.49	3

causes that might otherwise be conjectured.

RELATIONS TO OTHER CYCLES

Assuming that this stock price cycle is not the result of chance, is this length of $17.16 \pm .015$ weeks related simply to other wave lengths? Are there cycles 34.33 weeks long (17.16×2)? 51.5 weeks long (17.16×3)? Etc.?

When this work was started and the length of the cycle seemed to be 17.12 weeks (119.84 days) long, it was conjectured that the cycle might prove to be $1/54$ of the so-called 17 $3/4$ -year cycle. Fifty-four is a simple double and triple progression value ($3 \times 3 \times 3 \times 2$). Seventeen and three-quarter years is 6483 days. Fifty-four times 119.84 days is 6471.36 days. The agreement is rather close, especially as there is reason to believe that the length of 17.75 years is a shade too long.

However, 54 times 120.16 days is 6489 days. There is, therefore, no simple relationship between the two, unless the so-called 17.75-year wave turns out to be longer than now thought, or unless it has a varying length and the period 1897—1959 happens to coincide with one of its longer segments.

Are there other lengths that may be related? Table 4 shows multiples of 120.16 days by double and triple progression up to 25 years.

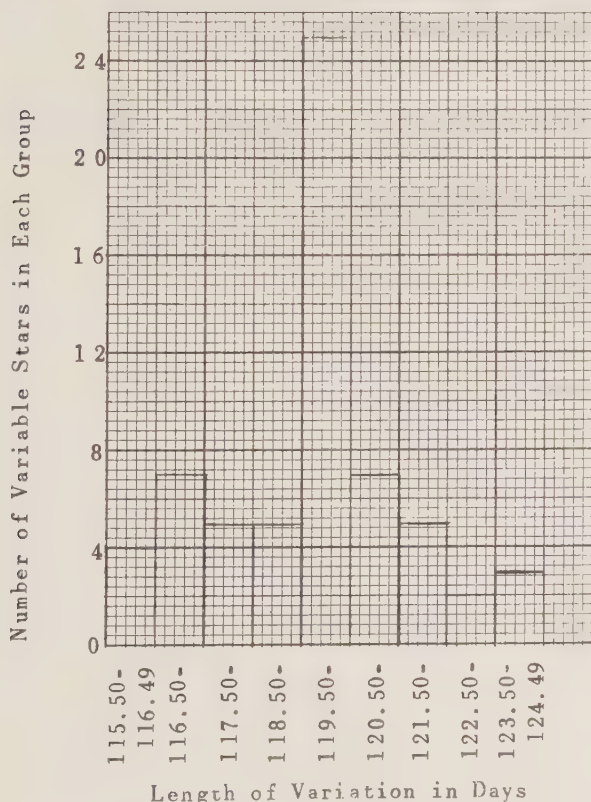


FIG. 9: NUMBER OF VARIABLE STARS
115.50 TO 124.49 DAY PERIODS

I fail to notice, among these lengths, any that are so well established and/or so accurately determined that a family relationship is suggested. These lengths should be kept in mind, however, in connection with future work. If, for example, the so-called 4-year cycle should, when measured more accurately, turn out to be 3.95 years long, we would have at least a relationship. Whether or not the relationship would be significant is, of course, another matter.

SUMMARY

We extended our study of the 17-week cycle in stock prices backward from 1928 to 1897. We found that it continued to be present, on the average, over a period of 31 additional years of time. We have thus greatly reduced the possibility that this behavior could be the result of chance.

We measured period or wave length of the cycle within an accuracy of about $1/10$ of 1%. This accuracy is sufficient to prove that it has no association with the sidereal periods of planets. This accuracy also dissociates this cycle, for the present, from any "family" relationship with other cycles.

Data are not at hand with which to test theories of possible solar relationship or theories of relationship to synodic periods

TABLE 4

The Period of 120.16 Days Multiplied
by Double and Triple Progression Factors
From One to 81, with Results Expressed
in Days, Months, and/or Years

Factor	In Days	Length Months	Years
1	120.16	3.95	.329007410
2	240.33	7.90	.658
3	360.50	11.84	.987
4	480.66	15.79	1.32
6	721.00	23.69	1.97
8		31.58	2.63
9		35.53	2.96
12		47.38	3.95
16		63.17	5.26
18		71.07	5.92
24		94.75	7.90
27			8.88
32			10.53
36			11.84
48			15.79
54			17.77
64			21.06
72			23.69
81			26.65

of the planets.

A curious concentration of variable star wave lengths at $17 \frac{1}{6}$ weeks is noted.

There is more than a suggestion that the $17 \frac{1}{6}$ -week cycle experiences a slight variation of wave length in a cycle which, on its part, is perhaps 23 or 24 years long.

A filtering of the raw deviations reinforces the conclusion reached from observation of the

raw deviations themselves that the proper model for the 17.166-week wave is not a perfectly regular periodicity, but rather a manifestation that occurs by fits and starts, sometimes longer than $17 \frac{1}{6}$ weeks, sometimes shorter. If there is a key to the variations in strength and timing, and if this key can be discovered, a great stride forward in cycle study will have been made.

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APPENDIX

TABLE A

Details of the Arithmetic
Main Tabulation Sheet

Week Number	Week Ending	A. The Data Weekly Average of Daily Close, As Published	B. Logs of the Data	C. Logs of the Data To July 30, 1914 Minus 0.139 To Make Them Comparable With Later Values	D. 17-Week Moving Average Of the Logs of the Data (Col. C to July 30, 1914, Col. B Thereafter)	E. Deviations, Logs of Data From Their 17-Week Moving Average (Col. C or Col. B Plus 2.000 Minus Col. D) Charted in Fig. 1	F. Simplified Deviations (Col. E Above and Below 2.000; Times 1,000)	G. 5-Section Moving Average of the 17.166-Week Periodic Table (From Table C)	H. 9-Week Moving Average of the Values in Col. G. Charted in Fig. 8
340	Jan. 9, 1897	40.81	1.610	1.471					
341	16	41.75	1.621	1.482					
342	23	42.62	1.630	1.491					
343	30	42.12	1.624	1.485					
344	Feb. 6	41.71	1.620	1.481					
345	13	40.45	1.607	1.468					
346	20	40.26	1.605	1.466					
347	27	40.93	1.612	1.473					
348	Mar. 6	41.63	1.619	1.480	1.471	2.009	9		
349	13	41.93	1.623	1.484	1.470	2.014	14		
350	20	41.58	1.619	1.480	1.469	2.011	11		
351	27	40.50	1.607	1.468	1.466	2.002	2		
352	Apr. 3	39.66	1.598	1.459	1.465	1.994	-6		
353	10	39.99	1.602	1.463	1.464	1.999	-1		
354	17	39.72	1.599	1.460	1.465	1.995	-5		
355	24	38.62	1.587	1.442	1.467	1.981	-19		
etc.									
381	23	49.82	1.697	1.558	1.563	1.995	-5		
382	30	48.79	1.688	1.549	1.561	1.988	-12	-3.0	
383	Nov. 6	47.23	1.674	1.535	1.558	1.977	-23	-7.2	
384	13	46.51	1.668	1.529	1.555	1.974	-26	-10.0	
385	20	46.97	1.672	1.533	1.553	1.980	-20	-11.4	
386	27	46.72	1.670	1.531	1.551	1.980	-20	-11.2	-4.6
387	Dec. 4	47.92	1.681	1.542	1.549	1.993	-7	-1.6	-3.4
388	11	49.31	1.693	1.554	1.548	2.006	6	2.4	-1.3
389	18	49.00	1.690	1.551	1.549	2.002	2	-0.2	0.8
390	25	48.64	1.687	1.548	1.547	2.000	0	0.6	2.9
391	Jan. 1, 1898	49.32	1.693	1.554	1.547	2.007	7	8.2	4.4
392	8	49.87	1.698	1.559	1.546	2.013	13	11.2	5.2
etc.									
3621	Nov. 27, 1898	649.99	2.813		2.815	1.998	-2		
3622	Dec. 4	662.36	2.821		2.815	2.006	6		
3623	11	671.11	2.827		2.815	2.012	12		
3624	18	674.92	2.829		2.814	2.015	15		
3625	25	672.15	2.827						
3626	Jan. 1, 1899	674.58	2.829						
3627	8	680.11	2.833						
3628	15	660.85	2.820						
3629	22	646.78	2.811						
3630	29	633.81	2.802						
3631	Feb. 5	630.40	2.800						
3632	12	622.41	2.794						

TABLE B

The 17.166-Week Periodic Table

Base Number	Weeks After Base																	
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	17.16
347	9	14	11	2	-6	-1	-5	-19	-16	-14	-15	-22	-20	-15	-2	-3	-6	
364	-3	-3	-9	0	-4	16	19	16	19	37	40	36	20	10	10	-12	-5	
381	-12	-23	-26	-20	-20	-7	6	2	0	7	13	15	8	10	18	22	15	-11
399	-4	-19	-19	-37	-20	-16	-17	-32	-29	8	18	14	18	21	20	2	-15	
416	-5	-8	-13	-18	-7	0	9	23	36	35	24	17	13	-13	-23	-34	-35	
433	-19	-12	-4	1	-3	0	7	4	-1	-1	0	0	-2	2	0	-19	-19	
450	2	-8	-9	5	12	25	24	23	27	29	17	-17	-16	-11	-20	-7	-2	
467	-15	-12	-3	-7	-2	-3	6	10	15	14	12	13	6	-5	-10	-16	-10	
484	-2	11	19	17	24	31	34	17	-22	-60	-25	-4	-15	-11	-6	11	10	14
502	4	-9	-17	-10	2	13	21	20	3	9	8	-2	-16	-10	0	-8	-12	
etc.																		
3574	7	8	5	2	-2	-9	-4	2	5	5	3	-2	-5	-5	-3	5	4	-1
3592	3	3	3	-1	-11	-11	-6	0	9	6	9	17	17	5	3	7	4	
3609	-6	-11	-14	-6	-6	-4	-8	-3	-1	-2	-8	-2	6	12	15			

Averages, by groups of 36, posted into Table 2.

TABLE C

Five-Section Moving Average of 17.166-Week Periodic Table

Base Number	Weeks After Base																	
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	17.16
347																		
364																		
381	-3.0	-7.2	-10.0	-11.4	-11.2	-1.6	2.4	-0.2	0.6	8.2	11.2	9.2	7.8	2.4	5.2	-2.6	-6.2	
399	-8.6	-12.4	-13.0	-11.4	-10.8	-1.4	4.8	4.4	3.6	10.8	14.2	13.2	11.4	5.8	5.6	-5.8	-8.8	
416	-7.6	-13.4	-13.0	-10.4	-7.6	-0.6	5.0	5.2	3.8	10.8	13.6	5.8	4.2	1.6	-0.4	-4.8	-8.2	
433	-8.2	-11.8	-9.6	-7.8	-5.2	0.2	5.0	6.8	6.8	12.2	13.4	5.4	3.8	-1.4	-6.0	-12.0	-13.2	
etc.																		
3540	2.6	1.0	2.0	1.0	-2.4	-5.4	-6.0	-2.2	-0.4	0.4	3.4	-0.8	-2.2	-4.4	-3.8	-0.6	2.2	
3557	1.4	2.4	1.8	2.4	-2.4	-3.8	-5.0	-0.4	3.4	2.4	4.6	1.0	1.0	-1.0	-1.4	1.4	2.6	
3574	0.0	-0.6	-2.2	-0.8	-3.4	-4.0	-4.4	-1.2	2.0	1.8	2.8	1.8	4.0	2.8	2.2			

The value of -3.0 for week 382 (one week after base 381) is the average of the first five values in Col. 1 of Table B, viz.: 9, -3, -12, -4, and -5. The value of -8.6 for week 400 (one week after base 399) is the average of the

five values in Col. 1 of Table B centering on that date, viz.: -3, -12, -4, -5, and -19, etc.

These values are now posted in Col. G of Table A.

PROGRAMMING CIRCULAR AUTOCORRELATION FOR CYCLE RESEARCH

BY B. G. HAVILL, JOHN E. PEARSON, AND STANLEY A. SELF*

The computational requirements for measuring cyclical fluctuations are so great that many useful statistical techniques have not been fully utilized. Seldom does the researcher of time-series fluctuations consider periods of data of less than fifty years and more often periods of one hundred years or more are selected for analysis. With a period of one hundred years or more the adjustments for seasonal variations, minor variations, and trend estimates often belabor the researcher to an extent that many detailed comparisons are neglected.

By using electronic data processing, a statistical technique such as autocorrelation may become a practical tool in cycle research. With the use of an electronic computer, intermediate or larger in size, the large number of coefficients of correlation may be computed and punched on output cards in a matter of minutes.

The product-moment correlation coefficient for the first order between successive terms X_1, X_2, \dots, X_n may be expressed as:

$$r_1 = \frac{\text{covariation } (X_j, X_{j+1})}{(\text{variation } X_j, \text{variation } X_{j+1})^{1/2}}$$

This coefficient is also referred to as the first serial correlation coefficient. Autocorrelation consists of all possible serial correlations. A general expression of k serial correlation coefficients for the terms X_1, X_2, \dots, X_n is:

$$r_k = \frac{\text{covariation } (X_j, X_{j+k})}{(\text{variation } X_j, \text{variation } X_{j+k})^{1/2}}$$

which is equal

$$\frac{1}{n-k} \sum_{j=1}^{n-k} (X_j X_{j+k}) - \frac{1}{(n-k)^2} \left(\sum_{j=1}^{n-k} X_j \right) \left(\sum_{j=1}^{n-k} X_{j+k} \right) \quad 1./$$

$$\left[\frac{1}{n-k} \sum_{j=1}^{n-k} X_j^2 - \frac{1}{(n-k)^2} \left(\sum_{j=1}^{n-k} X_j \right)^2 \right]^{1/2} \left[\frac{1}{n-k} \sum_{j=1}^{n-k} X_{j+k}^2 - \frac{1}{(n-k)^2} \left(\sum_{j=1}^{n-k} X_{j+k} \right)^2 \right]^{1/2}$$

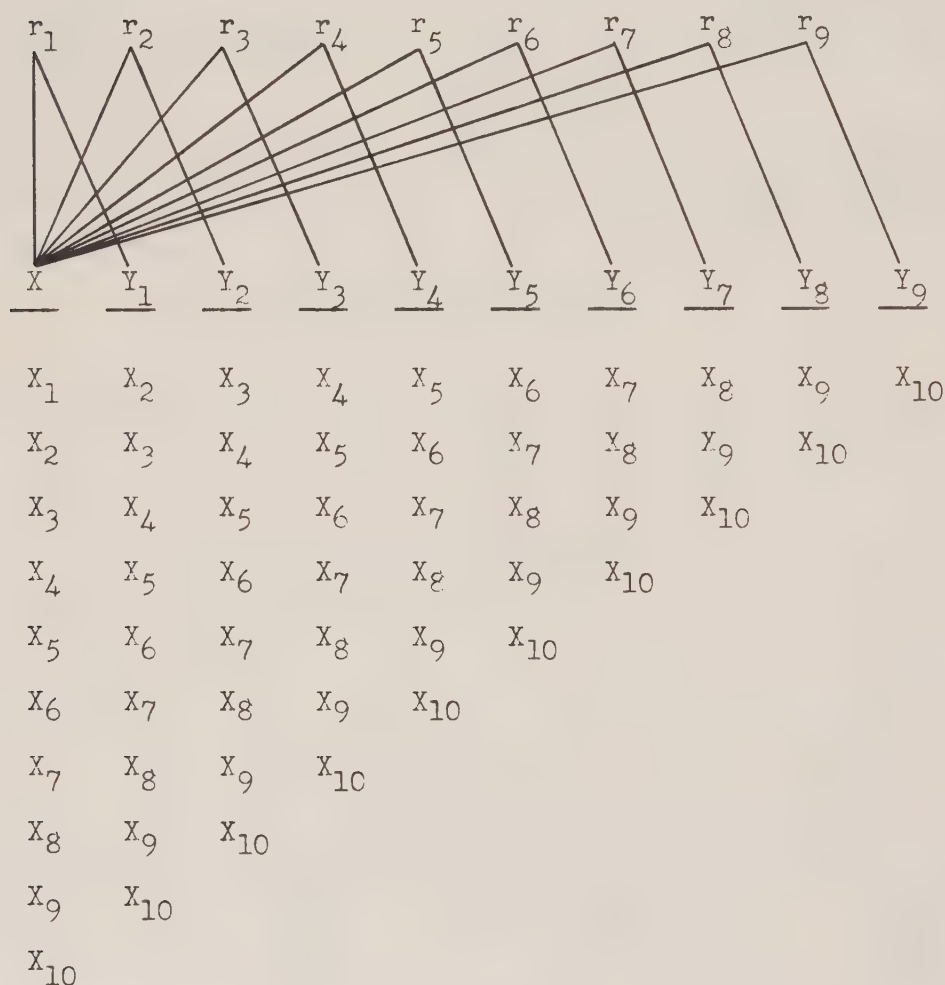
1./ Adapted from Maurice G. Kendall, *The Advanced Theory of Statistics*, (second edition 1948, Charles Griffin and Company Limited, London), p. 402.

*B. G. Havill is associated with Haskins and Sells, Public Accountants, Dallas, Texas. Dr. John E. Pearson is Associate Professor of Statistics and Chairman, Division of Business Services, and Dr. Stanley A. Self is Associate Professor of Management, both at the School of Business Administration, North Texas State College, Denton, Texas.

There are two general types of autocorrelation to which this formula might apply—linear and circular—and a distinction should be made between the two. Both types may be used in cycle research, but they are used for different purposes. Linear autocorrelation may be used to measure cyclical amplitudes and test sample independence whereas circular autocorrelation is useful in the measurement of cyclical durations.

Linear autocorrelation is a sequence of serial correlations where the lag between the items correlated continues to increase, and the number of terms decrease, until only the first and last items are correlated. The serial correlations include successively fewer items as they diminish until $n - 1$ coefficients are computed. This may be observed in Table 1 which shows the successive sets of variables from which the coefficients of correlation are computed.

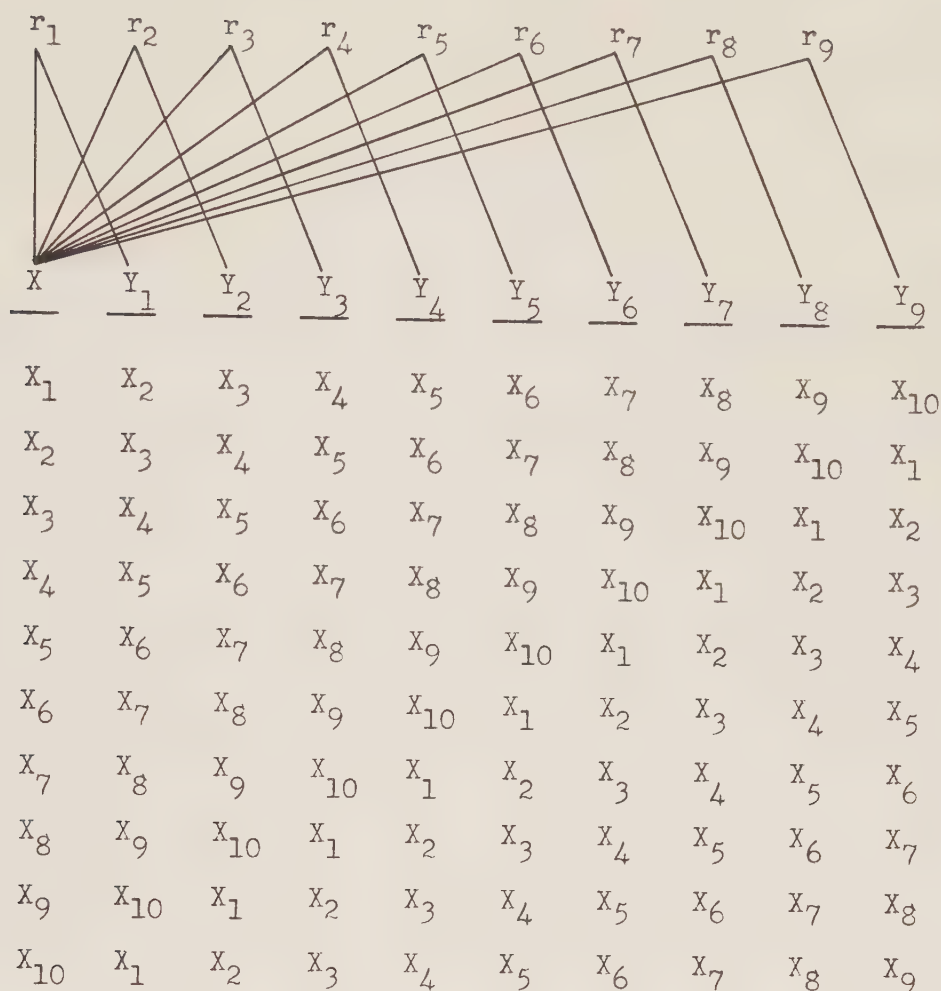
Table 1
Linear Autocorrelation



The electronic computation of linear autocorrelation is relatively slow because the change in the number of items for each coefficient leaves no constants in storage.

In regard to computation, circular autocorrelation differs from linear autocorrelation in that circular autocorrelation always includes the same number of items. The sum of each variable and the product of the sums of variables are constants also and only the sum of the cross-products of the variables is not a constant. This is true because the variables cut off at the beginning by the lagging procedure are reinstated at the end of that variable column. (See Table 2).

Table 2
Circular Autocorrelation



Consider the product-moment formula for computing simple linear correlation of two variables:

$$r = \frac{N \sum XY - \sum X \sum Y}{[N \sum X^2 - (\sum X)^2]^{\frac{1}{2}} [N \sum Y^2 - (\sum Y)^2]^{\frac{1}{2}}}$$

If $N \sum XY$ is the only variable, $\sum X \sum Y$ equals $(\sum X)^2$ equals $(\sum Y)^2$, and $N \sum X^2$ equals $N \sum Y^2$. Therefore:

$$r = \frac{N \sum XY - \sum X \sum Y}{[N \sum X^2 - (\sum X)^2]^{\frac{1}{2}} [N \sum Y^2 - (\sum Y)^2]^{\frac{1}{2}}} = \frac{N \sum XY - (\sum X)^2}{N \sum X^2 - (\sum X)^2}$$

Also, since $(\sum X)^2$ and N are constants, only $N \sum XY$ must be calculated for each coefficient.

The programming of circular autocorrelation on most computers involves two general steps: (A) calculate the constants, and (B) calculate the $N \sum XY$ variable and the coefficient of correlation for each X value increment. The latter step may be described in six parts: (1) the set-up operation, (2) the read operation, (3) calculating constants, (4) calculating $\sum XY$, (5) calculating the coefficient, and (6) the set-up operation for the next pass.

A more detailed description of these parts as they apply to programming circular auto-correlation on an IBM 650 may be given.

1. The Set-up Operation. The first word of the first card contains the number of items to follow and the machine sets up the program for this number. If part of the program has been run previously and some of the coefficients have already been calculated, the program can easily begin in the middle through the use of the second word in the set-up card. To begin in the middle of the problem, the increment of the *last* coefficient processed must be punched in the second word of the set-up card and the rest of the coefficients are calculated.

2. The Read Operation. The individual X values are read from any column of cards and the values are stored into drum locations. Also, as each X value is read and stored the values of the data are totaled, so $\sum X$ is obtained at the end of the read operation. Also, as each X value is being stored, the values of X^2 are calculated and $\sum X^2$ is accumulated.

3. Calculate Constants. As was mentioned, each factor of the coefficient of correlation formula is constant with the exception of $N\sum XY$. After the last card is read, the program calculates $(\sum X)^2$ and $N\sum X^2 - (\sum X)^2$, and since $\sum X$ and $\sum X^2$ have already been accumulated, all constants are stored for later use.

4. Calculate $\sum XY$. The machine first analyzes the number of shifts which have previously been made, and if that number is equal to the number of items of data, the program is finished and the machine stops. If the number of shifts is less than the number of items of data, the machine proceeds to calculate $\sum XY$. In order to calculate any given XY value, the machine must detect which X value to multiply, and select the appropriate Y value.²

On the first pass through the program X_1 is multiplied by Y_1 ; X_2 by Y_2 etc. until X_n is multiplied by Y_n . As these X and Y values are multiplied together, the products are summed, so that when the last X value is multiplied by the last Y value, the total is $\sum XY$. The program then proceeds through the further steps and calculates the first coefficient, sets up the program for the second pass, and returns to this part of the program.

On the second pass, X_1 is multiplied by Y_2 ; X_2 by Y_3 . . . X_{n-1} by Y_n ; and X_n by Y_1 . The program completes the calculations for the coefficients, sets up the program for the next pass, and returns.

Subsequent passes are similar to the second and the last pass multiplies X_1 by Y_n ; X_2 by Y_1 ; . . . X_n by Y_{n-1} .

The process is similar to writing the X values on two cylinders, lining X_1 with X_1 , multiplying each of the X values around on the two cylinders together, and then turning one cylinder one value and repeating the process until the first value on the stationary cylinder is multiplied by the last value on the moving cylinder. The next shift would mean that the process was starting over again; likewise, when N number of shifts have been made, the program stops.

5. Calculate Coefficient. After $\sum XY$ has been calculated the machine is ready to calculate the coefficient. It first multiplies N times $\sum XY$, then subtracts $(\sum X)^2$, and divides by the constant denominator previously computed. The coefficient and number of the shift, or increment value, is then punched.

6. The Set-up Operation for the Next Pass. After one coefficient of correlation is calculated, the machine clears the storage of $\sum XY$, readies itself to multiply X_1 by a Y value which it determines by adding the value 1 to the storage location of the *first* Y value used in the previous pass. The program then returns to the beginning of the "Calculate $\sum XY$ " operation.

By using electronic computation an otherwise cumbersome and tedious technique is reduced to a deck of 119 instruction cards (for the IBM 650) which requires only a few minutes for completed computations. The opportunities to use the technique in cycle research are many, and since it is easily programmed and computed on electronic computers, it should be used more extensively in the future.

²There actually are no Y values since there is only one series of chronologically classified data. However, it is assumed that: $X_1 = Y_1$; $X_2 = Y_2$; . . . $X_n = Y_n$ for simplicity in keeping X^2 and XY separate.

THE CALENDAR TIMING OF VARIOUS HELIOCENTRIC LONGITUDES OF JUPITER JANUARY 1897 — DECEMBER 1959

BY RUTH V. MURTHA

As the planets travel in ellipses they travel at different speeds in different parts of their orbits. Consequently, equal heliocentric angles relative to the stars and relative to each other occur at unequal time intervals.

To test the theory that certain terrestrial cycles are associated with equal angular planetary positions, relative to the stars or relative to each other, one must know the position of the various planets at various times. In reverse, one must know the calendar times at which the planets are at various stipulated positions.

This information is readily available in the successive annual volumes of the *American Ephemeris and Nautical Almanac*, either by reference or by interpolation.

This paper will provide this information, thus derived, for the planet Jupiter at approximately 4-month intervals for the period Jan-

uary 1, 1897 through December 31, 1959.

On the average it takes Jupiter 4,332.5870 days (11.86 years) to go once around the sun relative to the stars. The time from perihelion to perihelion is about 1/1,000 of a day more.

Perihelion is the position closest to the sun, at which time the planet is moving fastest; aphelion is the position farthest from the sun at which time the planet moves slowest.

The actual time from aphelion to perihelion, or perihelion to perihelion, for the period 1897—1959 is given in Table 1 below. It is clear that, for any one revolution, the actual time varies slightly from the average time. These variations are due to the gravitational influence of the other planets. They are called perturbations.

As Baker puts it, "Thus far we have dealt with the revolution of a planet around the sun as though the planet were acted on only

TABLE 1

The Calendar Times of Various Aphelia and Perihelia of Jupiter, 1897—1959,
and the Time Intervals Between Them

Position	Time (Greenwich Civil Time)						Interval	
	Year	Month	Day	Hour	Julian Day	Hour	Days	Hours
Aphelion	1898	Jun	27	7	2,414,468	7	—	—
Perihelion	1904	Jun	2	0	2,416,634	0	2,165	17
Perihelion	1916	Apr	18	1	2,420,972	1	4,338	1
Perihelion	1928	Mar	15	7	2,425,321	7	4,349	6
Perihelion	1940	Jan	23	10	2,429,652	10	4,331	3
Perihelion	1951	Nov	21	7	2,433,972	7	4,319	21
Aphelion	1957	Oct	23	5	2,436,135	5	2,162	22
Mean (ideal), 1/2 revolution, in days							2,166.2935	
Mean (ideal), one revolution, in days							4,332.5870	

by the sun's attraction. This is the problem of two bodies, which is solved directly and completely in terms of the law of gravitation. But the planet is subject to the attractions of all the other members of the solar system as well, so that it departs in a complex manner from simple elliptic motion. Thus we have in practice the problem of three or more bodies, whose solution is much more troublesome. . .

"Perturbations are the alterations so produced. As examples, the eccentricities and inclinations of planetary orbits fluctuate, *perihelia advance* and *nodes regress*. All perturbations are oscillatory in the long run, so that they are not likely to permanently alter the arrangement of the solar system."¹

The advance of the perihelia is a permanent motion but, after millions of years, the perihelia come back to the same longitude. The same is true of the regression of the nodes.

If Jupiter had no perturbations, it would return to the same longitude at each perihelion after a revolution of 360° ; would travel from aphelion to perihelion in 180° . In view of the perturbations this is not strictly so. The actual facts are given in Table 2 which shows the longitude at perihelion (or aphelion), the number of degrees for each revolution (or half-revolution), and the number of degrees per $1/36$ th revolution. One thirty-sixth of a rev-

olution gives us a convenient number of points for plotting (about 3 a year, on the average).

By starting with the longitude of the perihelion of 1904, $12^\circ 25' 18.06''$, and adding increments of $10^\circ 1' 1.41''$, we get 36 equally spaced longitudes between June 2, 1904 and April 18, 1916, the next perihelion. The longitudes prior to June 2, 1904 and after November 21, 1951 are computed similarly assuming that aphelion represents one half of a revolution.

We can now find the days corresponding to each of these longitudes and the time intervals between each of these days. These various values are given in Table 3.

This information tells us quite closely how the length of a cycle would vary if it were related to the varying speed of Jupiter. It also tells us when the cycle would be long, when short. We see, for example, that a cycle that averaged $1/36$ of the average sidereal period of Jupiter (120.35 days) and which varied in length with the speed of Jupiter, would vary in length from about 109 days at perihelion to about 132 days at aphelion, with slight differences in successive revolutions.

Similarly a 240-day cycle associated with the angular variation of Jupiter would vary in length over the 11.86-year period from 218 to 264 days, and so on, and the long and the short waves would come at the indicated times. And so on for other lengths in which one might be interested.

¹Baker, Robert H. *Astronomy*. 5th Edition. Van Nostrand, 1950. p. 187.

TABLE 2

The Longitudes of Various Aphelia and Perihelia of Jupiter, 1879—1959,
the Degree Intervals Between Them, and $1/36$ th of the Degree Intervals Between Them

Position	Longitude			Interval 1 Revolution			Interval $1/36$ Revolution		
	o	'	''	o	'	''	o	'	''
Aphelion	192	25	48.52						
Perihelion	12	25	18.06	*179	59	29.54	9	59	58.31
Perihelion	13	2	8.74	360	36	50.68	10	1	1.41
Perihelion	14	30	53.07	361	28	44.33	10	2	27.89
Perihelion	14	33	10.48	360	2	17.41	10		3.82
Perihelion	13	39	5.49	359	5	55.01	9	58	29.89
Aphelion	193	28	23.99	*179	49	18.50	9	59	24.36

* $1/2$ revolution

TABLE 3

Various Successive Positions of Jupiter, 1897—1959,
the Longitudes at the Given Positions, the Corresponding Calendar and Julian Days,
and the Intervals in Days Between These Positions

Position (Perihelion plus fraction of revolution indicated)	Longitude at the given position			Corresponding Day				Interval in Days
				Calendar Day			Julian Day	
	°	'	"	Year	Month	Day	(2,4 omitted)	
14/36th	152	25	55.28	1897	Jan	18	13,493	
15/36th	162	25	53.59		May	28	14,073	130
16/36th	172	25	51.90		Oct	6	14,204	131
17/36th	182	25	50.21	1898	Feb	15	14,336	132
18/36th	192	25	48.52		Jun	27	14,468	132
(Aphelion)								
19/36th	202	25	46.83		Nov	6	14,600	132
20/36th	212	25	45.14	1899	Mar	18	14,732	132
21/36th	222	25	43.44		Jul	27	14,863	131
22/36th	232	25	41.75		Dec	4	14,993	130
23/36th	242	25	40.06	1900	Apr	11	15,121	128
24/36th	252	25	38.37		Aug	16	15,248	127
25/36th	262	25	36.67		Dec	19	15,373	125
26/36th	272	25	34.98	1901	Apr	21	15,496	123
27/36th	282	25	33.29		Aug	20	15,617	121
28/36th	292	25	31.60		Dec	17	15,736	119
29/36th	302	25	29.91	1902	Apr	13	15,853	117
30/36th	312	25	28.21		Aug	6	15,968	115
31/36th	322	25	26.52		Nov	28	16,082	114
32/36th	332	25	24.83	1903	Mar	20	16,194	112
33/36th	342	25	23.14		Jul	9	16,305	111
34/36th	352	25	21.44		Oct	27	16,415	110
35/36th	2	25	19.75	1904	Feb	14	16,525	110
36/36th	12	25	18.06		Jun	2	16,634	109
(Perihelion)								
1/36th	22	26	19.47		Sep	19	16,743	109
2/36th	32	27	20.88	1905	Jan	7	16,853	110
3/36th	42	28	22.28		Apr	27	16,963	110
4/36th	52	29	23.69		Aug	16	17,074	111
5/36th	62	30	25.10		Dec	6	17,186	112
6/36th	72	31	26.51	1906	Mar	29	17,299	113
7/36th	82	32	27.91		Jul	23	17,415	116
8/36th	92	33	29.32		Nov	17	17,532	117
9/36th	102	34	30.73	1907	Mar	16	17,651	119
10/36th	112	35	32.14		Jul	16	17,773	122
11/36th	122	36	33.55		Nov	16	17,896	123
12/36th	132	37	34.95	1908	Mar	20	18,021	125
13/36th	142	38	36.36		Jul	25	18,148	127
14/36th	152	39	37.77		Nov	30	18,276	128
15/36th	162	40	39.18	1909	Apr	9	18,406	130
16/36th	172	41	40.58		Aug	18	18,537	131
17/36th	182	42	41.99		Dec	28	18,669	132
18/36th	192	43	43.40	1910	May	10	18,802	133
19/36th	202	44	44.81		Sep	19	18,934	132
20/36th	212	45	46.22	1911	Jan	29	19,066	132
21/36th	222	46	47.62		Jun	10	19,198	132

TABLE 3 (Continued)

Position (Perihelion plus fraction of revolution indicated)	Longitude at the given position			Corresponding Day				Interval in Days
				Calendar Day			Julian Day	
	o	'	”	Year	Month	Day	(2,4 omitted)	
22/36th	232	47	49.03	1912	Oct	18	19,328	130
23/36th	242	48	50.44		Feb	24	19,457	129
24/36th	252	49	51.85		Jun	30	19,584	127
25/36th	262	50	53.25	1913	Nov	2	19,709	125
26/36th	272	51	54.66		Mar	5	19,832	123
27/36th	282	52	56.07		Jul	4	19,953	121
28/36th	292	53	57.48	1914	Oct	31	20,072	119
29/36th	302	54	58.89		Feb	26	20,190	118
30/36th	312	56	.29		Jun	21	20,305	115
31/36th	322	57	1.70	1915	Oct	13	20,419	114
32/36th	332	58	3.11		Feb	3	20,532	113
33/36th	342	59	4.52		May	25	20,643	111
34/36th	353	0	5.92	1916	Sep	12	20,753	110
35/36th	3	1	7.33		Dec	31	20,863	110
36/36th	13	2	8.74		Apr	18	20,972	109
(Perihelion)								
1/36th	23	4	36.64	1917	Aug	6	21,082	110
2/36th	33	7	4.54		Nov	24	21,192	110
3/36th	43	9	32.43		Mar	14	21,302	110
4/36th	53	12	0.33	1918	Jul	3	21,413	111
5/36th	63	14	28.23		Oct	24	21,526	113
6/36th	73	16	56.13		Feb	15	21,640	114
7/36th	83	19	24.03	1919	Jun	10	21,755	115
8/36th	93	21	51.92		Oct	6	21,873	118
9/36th	103	24	19.82		Feb	2	21,993	120
10/36th	113	26	47.72	1920	Jun	4	22,214	121
11/36th	123	29	15.62		Oct	5	22,237	123
12/36th	133	31	43.52		Feb	8	22,363	126
13/36th	143	34	11.41	1921	Jun	14	22,490	127
14/36th	153	36	39.31		Oct	21	22,619	129
15/36th	163	39	7.21		Feb	28	22,750	131
16/36th	173	41	35.11	1922	Jul	10	22,881	131
17/36th	183	44	3.01		Nov	19	23,013	132
18/36th	193	46	30.90		Apr	1	23,146	133
19/36th	203	48	58.80	1923	Aug	11	23,278	132
20/36th	213	51	26.70		Dec	22	23,411	133
21/36th	223	53	54.60		May	2	23,542	131
22/36th	233	56	22.50	1924	Sep	10	23,673	131
23/36th	243	58	50.40		Jan	17	23,802	129
24/36th	254	1	18.29		May	24	23,930	128
25/36th	264	3	46.19	1925	Sep	26	24,055	125
26/36th	274	6	14.09		Jan	28	24,179	124
27/36th	284	8	41.99		May	30	24,301	122
28/36th	294	11	9.89	1926	Sep	26	24,420	119
29/36th	304	13	37.78		Jan	22	24,538	118
30/36th	314	16	5.68		May	18	24,654	116
31/36th	324	18	33.58	1927	Sep	9	24,768	114
32/36th	334	21	1.48		Dec	30	24,880	112
33/36th	344	23	29.38		Apr	20	24,991	111
34/36th	354	25	57.27	1927	Aug	9	25,102	111
35/36th	4	28	25.17		Nov	27	25,212	110

TABLE 3 (Continued)

Position (Perihelion plus fraction of revolution indicated)	Longitude at the given position			Corresponding Day				Interval in Days
				Calendar Day			Julian Day	
	°	'	″	Year	Month	Day	(2,4 omitted)	
36/36th (Perihelion)	14	30	53.07	1928	Mar	16	25,322	110
1/36th	24	30	56.89		Jul	2	25,430	108
2/36th	34	31	.70		Oct	20	25,540	110
3/36th	44	31	4.52	1929	Feb	7	25,650	110
4/36th	54	31	8.34		May	29	25,761	111
5/36th	64	31	12.15		Sep	18	25,873	112
6/36th	74	31	15.97	1930	Jan	10	25,987	114
7/36th	84	31	19.79		May	5	26,102	115
8/36th	94	31	23.61		Aug	30	26,219	117
9/36th	104	31	27.42		Dec	26	26,338	119
10/36th	114	31	31.24	1931	Apr	26	26,458	120
11/36th	124	31	35.06		Aug	27	26,581	123
12/36th	134	31	38.87		Dec	30	26,706	125
13/36th	144	31	42.69	1932	May	5	26,833	127
14/36th	154	31	46.51		Sep	11	26,962	129
15/36th	164	31	50.32	1933	Jan	19	27,092	130
16/36th	174	31	54.14		May	30	27,223	131
17/36th	184	31	57.96		Oct	9	27,355	132
18/36th	194	32	1.77	1934	Feb	18	27,487	132
19/36th	204	32	5.59		Jun	30	27,619	132
20/36th	214	32	9.41		Nov	9	27,751	132
21/36th	224	32	13.22	1935	Mar	20	27,882	131
22/36th	234	32	17.04		Jul	28	28,012	130
23/36th	244	32	20.86		Dec	4	28,141	129
24/36th	254	32	24.68	1936	Apr	9	28,268	127
25/36th	264	32	28.49		Aug	12	28,393	125
26/36th	274	32	32.31		Dec	13	28,516	123
27/36th	284	32	36.13	1937	Apr	13	28,637	121
28/36th	294	32	39.94		Aug	10	28,756	119
29/36th	304	32	43.76		Dec	5	28,873	117
30/36th	314	32	47.58	1938	Mar	30	28,988	115
31/36th	324	32	51.39		Jul	21	29,101	113
32/36th	334	32	55.21		Nov	11	29,214	113
33/36th	344	32	59.03	1939	Mar	1	29,324	110
34/36th	354	33	2.84		Jun	19	29,434	110
35/36th	4	33	6.66		Oct	7	29,544	110
36/36th (Perihelion)	14	33	10.48	1940	Jan	23	29,652	108
1/36th	24	31	40.34		May	12	29,762	110
2/36th	34	30	10.20		Aug	28	29,870	108
3/36th	44	28	40.06		Dec	16	29,980	110
4/36th	54	27	9.93	1941	Apr	5	30,090	110
5/36th	64	25	39.79		Jul	26	30,202	112
6/36th	74	24	9.65		Nov	16	30,315	113
7/36th	84	22	39.51	1942	Mar	11	30,430	115
8/36th	94	21	9.37		Jul	6	30,547	117
9/36th	104	19	39.23		Nov	2	30,666	119
10/36th	114	18	9.09	1943	Mar	2	30,786	120
11/36th	124	16	38.96		Jul	3	30,909	123
12/36th	134	15	8.82		Nov	4	31,033	124

TABLE 3 (Continued)

Position (Perihelion plus fraction of revolution indicated)	Longitude at the given position			Corresponding Day				Interval in Days
				Calendar Day			Julian Day	
	°	'	″	Year	Month	Day	(2,4 omitted)	
13/36th	144	13	38.68	1944	Mar	10	31,160	127
14/36th	154	12	8.54		Jul	16	31,288	128
15/36th	164	10	38.40		Nov	23	31,418	130
16/36th	174	9	8.26	1945	Apr	3	31,549	131
17/36th	184	7	38.12		Aug	13	31,681	132
18/36th	194	6	7.99		Dec.	23	31,813	132
19/36th	204	4	37.85	1946	May	4	31,945	132
20/36th	214	3	7.71		Sep	13	32,077	132
21/36th	224	1	37.57	1947	Jan	22	32,208	131
22/36th	234	0	7.43		May	31	32,337	129
23/36th	243	58	37.29		Oct	6	32,465	128
24/36th	253	57	7.15	1948	Feb	10	32,592	127
25/36th	263	55	37.02		Jun	14	32,717	125
26/36th	273	54	6.88		Oct	14	32,839	122
27/36th	283	52	36.74	1949	Feb	12	32,960	121
28/36th	293	51	6.60		Jun	10	33,078	118
29/36th	303	49	36.46		Oct	5	33,195	117
30/36th	313	48	6.32	1950	Jan	28	33,310	115
31/36th	323	46	36.18		May	21	33,423	113
32/36th	333	45	6.04		Sep	10	33,535	112
33/36th	343	43	35.91	1950	Dec	29	33,645	110
34/36th	353	42	5.77	1951	Apr	18	33,755	110
35/36th	3	40	35.62		Aug	5	33,864	109
36/36th	13	39	5.49		Nov	22	33,973	109
(Perihelion)								
1/36th	23	38	29.85	1952	Mar	9	34,081	108
2/36th	33	37	54.21		Jun	26	34,189	108
3/36th	43	37	18.57		Oct	14	34,299	110
4/36th	53	36	42.93	1953	Feb	2	34,411	112
5/36th	63	36	7.30		May	25	34,523	112
6/36th	73	35	31.66		Sep	15	34,636	113
7/36th	83	34	56.02	1954	Jan	8	34,751	115
8/36th	93	34	20.38		May	5	34,868	117
9/36th	103	33	44.74		Sep	1	34,987	119
10/36th	113	33	9.10		Dec	30	35,107	120
11/36th	123	32	33.46	1955	May	2	35,230	123
12/36th	133	31	57.82		Sep	4	35,355	125
13/36th	143	31	22.19	1956	Jan	8	35,481	126
14/36th	153	30	46.55		May	15	35,609	128
15/36th	163	30	10.91		Sep	23	35,740	131
16/36th	173	29	35.27	1957	Feb	1	35,872	132
17/36th	183	28	59.63		Jun	13	36,003	131
18/36th	193	28	23.99		Oct	23	36,135	132
(Aphelion)								
19/36th	203	27	48.35	1958	Mar	4	36,267	132
20/36th	213	27	12.71		Jul	14	36,399	132
21/36th	223	26	37.07		Nov	22	36,530	131
22/36th	233	26	1.43	1959	Apr	1	36,660	130
23/36th	243	25	25.79		Aug	7	36,788	128
24/36th	253	24	50.15		Dec	12	36,915	127

APPENDIX

DAY NUMBER AND DECIMAL EQUIVALENT

(NON-LEAP-YEARS ONLY)

JANUARY			MARCH			MAY			JULY			SEPTEMBER			NOVEMBER		
1	1	.003	1	60	.164	1	121	.332	1	182	.499	1	244	.668	1	305	.836
2	2	.005	2	61	.167	2	122	.334	2	183	.501	2	245	.671	2	306	.838
3	3	.008	3	62	.170	3	123	.337	3	184	.504	3	246	.674	3	307	.841
4	4	.011	4	63	.173	4	124	.340	4	185	.507	4	247	.677	4	308	.844
5	5	.014	5	64	.175	5	125	.342	5	186	.510	5	248	.679	5	309	.847
6	6	.016	6	65	.178	6	126	.345	6	187	.512	6	249	.682	6	310	.849
7	7	.019	7	66	.181	7	127	.348	7	188	.515	7	250	.685	7	311	.852
8	8	.022	8	67	.184	8	128	.351	8	189	.518	8	251	.688	8	312	.855
9	9	.025	9	68	.186	9	129	.353	9	190	.521	9	252	.690	9	313	.858
10	10	.027	10	69	.189	10	130	.356	10	191	.523	10	253	.693	10	314	.860
11	11	.030	11	70	.192	11	131	.359	11	192	.526	11	254	.696	11	315	.863
12	12	.033	12	71	.195	12	132	.362	12	193	.529	12	255	.699	12	316	.866
13	13	.036	13	72	.197	13	133	.364	13	194	.532	13	256	.701	13	317	.868
14	14	.038	14	73	.200	14	134	.367	14	195	.534	14	257	.704	14	318	.871
15	15	.041	15	74	.203	15	135	.370	15	196	.537	15	258	.707	15	319	.874
16	16	.044	16	75	.205	16	136	.373	16	197	.540	16	259	.710	16	320	.877
17	17	.047	17	76	.208	17	137	.375	17	198	.542	17	260	.712	17	321	.879
18	18	.049	18	77	.211	18	138	.378	18	199	.545	18	261	.715	18	322	.882
19	19	.052	19	78	.214	19	139	.381	19	200	.548	19	262	.718	19	323	.885
20	20	.055	20	79	.216	20	140	.384	20	201	.551	20	263	.721	20	324	.888
21	21	.058	21	80	.219	21	141	.386	21	202	.553	21	264	.723	21	325	.890
22	22	.063	22	81	.222	22	142	.389	22	203	.556	22	265	.726	22	326	.893
23	23	.063	23	82	.225	23	143	.392	23	204	.559	23	266	.729	23	327	.896
24	24	.066	24	83	.227	24	144	.395	24	205	.562	24	267	.732	24	328	.899
25	25	.068	25	84	.230	25	145	.397	25	206	.564	25	268	.734	25	329	.901
26	26	.071	26	85	.233	26	146	.400	26	207	.567	26	269	.737	26	330	.904
27	27	.074	27	86	.236	27	147	.403	27	208	.570	27	270	.740	27	331	.907
28	28	.077	28	87	.238	28	148	.405	28	209	.573	28	271	.742	28	332	.910
29	29	.079	29	88	.241	29	149	.408	29	210	.575	29	272	.745	29	333	.912
30	30	.082	30	89	.244	30	150	.411	30	211	.578	30	273	.748	30	334	.915
31	31	.085	31	90	.247	31	151	.414	31	212	.581	OCTOBER			DECEMBER		
FEBRUARY			APRIL			JUNE			AUGUST			1	274	.751	1	335	.918
1	32	.088	1	91	.249	1	152	.416	1	213	.584	2	275	.753	2	336	.921
2	33	.090	2	92	.252	2	153	.419	2	214	.586	3	276	.756	3	337	.923
3	34	.093	3	93	.255	3	154	.422	3	215	.589	4	277	.759	4	338	.926
4	35	.096	4	94	.258	4	155	.425	4	216	.592	5	278	.762	5	339	.929
5	36	.099	5	95	.260	5	156	.427	5	217	.595	6	279	.764	6	340	.932
6	37	.101	6	96	.263	6	157	.430	6	218	.598	7	280	.767	7	341	.934
7	38	.104	7	97	.266	7	158	.433	7	219	.600	8	281	.770	8	342	.937
8	39	.107	8	98	.268	8	159	.436	8	220	.603	9	282	.773	9	343	.940
9	40	.110	9	99	.271	9	160	.438	9	221	.605	10	283	.775	10	344	.942
10	41	.112	10	100	.274	10	161	.441	10	222	.608	11	284	.778	11	345	.945
11	42	.115	11	101	.277	11	162	.444	11	223	.611	12	285	.781	12	346	.948
12	43	.118	12	102	.279	12	163	.447	12	224	.614	13	286	.784	13	347	.951
13	44	.121	13	103	.282	13	164	.449	13	225	.616	14	287	.786	14	348	.953
14	45	.123	14	104	.285	14	165	.452	14	226	.619	15	288	.789	15	349	.956
15	46	.126	15	105	.288	15	166	.455	15	227	.622	16	289	.792	16	350	.959
16	47	.129	16	106	.290	16	167	.458	16	228	.625	17	290	.795	17	351	.962
17	48	.132	17	107	.293	17	168	.460	17	229	.627	18	291	.797	18	352	.964
18	49	.134	18	108	.296	18	169	.463	18	230	.630	19	292	.800	19	353	.967
19	50	.137	19	109	.299	19	170	.466	19	231	.633	20	293	.803	20	354	.970
20	51	.140	20	110	.301	20	171	.468	20	232	.636	21	294	.805	21	355	.973
21	52	.143	21	111	.304	21	172	.471	21	233	.638	22	295	.808	22	356	.975
22	53	.145	22	112	.307	22	173	.474	22	234	.641	23	296	.811	23	357	.978
23	54	.148	23	113	.310	23	174	.477	23	235	.644	24	297	.814	24	358	.981
24	55	.151	24	114	.312	24	175	.479	24	236	.647	25	298	.816	25	359	.984
25	56	.153	25	115	.315	25	176	.482	25	237	.649	26	299	.819	26	360	.986
26	57	.156	26	116	.318	26	177	.485	26	238	.652	27	300	.822	27	361	.989
27	58	.159	27	117	.321	27	178	.488	27	239	.655	28	301	.825	28	362	.992
28	59	.162	28	118	.323	28	179	.490	28	240	.658	29	302	.827	29	363	.995
			29	119	.326	29	180	.493	29	241	.660	30	303	.830	30	364	.997
			30	120	.329	30	181	.496	30	242	.663	31	304	.833	31	365	1.000
									31	243	.666						

